Introduction

The test of comprehension is one of the most important areas of evaluation of the aptitude of the candidate. It assesses a person's ability to understand an issue, analyse it and distinguish what is relevant and what is not. This test usually comprises a comprehension passage followed by a set of questions. The candidate is required to read and comprehend the passage and answer the ensuing questions which are multiple choice objective type. The nature of the questions are such that they test not merely the candidate's ability of literal understanding of what is written, but more importantly, ability to fully comprehend the issue dealt with, to apply his own thought to the subject and to take a decision as to what should be the most appropriate choice in the given context.

1 Features of Comprehension Passages

The comprehension passages will incorporate subjects of different nature. These may range from issues like governance to global warming, bureaucratic accountability to demographic development and profile, from economic concepts to scientific researches, from programmes of local government to policies of international organisations. But notwithstanding the range of issues, the nature of the passage will be such as not to require a prior indepth knowledge of the subject dealt therein. Implying thereby that even a candidate having little previous exposure to the issue, but having the basic comprehension ability, will well be able to understand the passage and attempt the questions. The length of the passage is likely to be of 200 to 300 words. Depending on the length and nature of the passage, the candidate will be required to answer three to five questions from each passage. Each of these questions is likely to be of two and a half marks. This section (on comprehension) is likely to have around 25% to 30% weightage in the entire question paper. As such, expect five to six comprehension passages, carrying anything between 60 to 65 marks.

2 Essential Abilities

Primarily the candidates are required to have the ability to comprehend the passage, understand the importance of the question and select the appropriate choice. But reducing the skills necessary to these three broad categories, will perhaps be too simplistic. Each of these abilities, in turn, requires several skills and abilities, the important ones of which are being discussed below.

2.1 Careful Reading

The reading of the comprehension passage has to be careful and meticulous. Unlike the perusal of storybooks, magazines, and newspapers where one tends to go through the matter rather cursorily, or at best casually, comprehension passages require very focussed and thorough reading.

Every sentence and every word has to be meticulously gone through and what the author is trying to say has to be grasped. There are times when, while reading in ordinary course, we are not able to fully comprehend what is implied in a sentence or in a paragraph. Although we have visually perused such a portion, we choose to move ahead without waiting to completely understand what has been said or without making a diligent effort to do so. Such an approach is likely to backfire in the test of comprehension ability.

As a result of improper comprehension, either we may not be able to pick the correct choice or lose valuable time in re-reading the relevant portion and then deciding the choice of answer. So, careful and focused reading has to be made your second nature.

2.2 Comprehension of Language

Comprehension of language means, having the ability of literal understanding of the passage. This requires a basic command over the language and a reasonable vocabulary. Although the degree of proficiency of linguistic skills is put

to test in the section on,'English Language Comprehension Skills', good vocabulary and command over language is sure to aid in quicker and better grasp of the passage. This is specially true about your vocabulary. If you do not happen to know the meaning of a word used in the passage, then it is possible that your understanding of what the word implies is only a contextual conjecture, which may or may not be right. So, if you are not hard pressed for time, working on your language skills is likely to be doubly helpful.

2.3 Ability to Concentrate

Concentration is one ability which is of utmost importance in the entire examination and more so in this section. Even a momentary lapse of concentration can mean that you lose grasp of the subject or waste valuable time in again having to go through the portion where your concentration wavered. And this occasionally does happen even with the best and the most alert readers. While the eyes keep on perusing the text, the mind loses connect with the subject and by the time we become aware of the concentration lapse, we have already marched ahead to another line or a paragraph. And then comes the quandary. Whether to keep on reading ahead and risk not understanding the passage fully, or whether to return back to the portion where we mentally strayed off and spend more time in grasping it. While the first choice is frought with the possibility of loss of marks due to improper comprehension, the second one entails loss of precious time. In either case what is adversely affected, is the candidate's performance.

Thus it is essential to fully concentrate on the passage, to be aware of the occurrence of concentration lapse (if that happens) and to come back to course as quickly as possible.

2.4 Visualising What is Being Said

The previous section dealt with the significance and need for rapt concentration while going through a comprehension passage. While the necessity for concentration can hardly be over-emphasised, the significant question is how to achieve it or improve upon it. An important skill which prevents the concentration from wavering is the ability to visualise what is being said. In simple words, this means trying to picturise the letter and import of the text. This is 'converse of imagery', where vivid descriptions in words are used to produce a mental image. In fact visualising what is being said or written is a usual feature of our brain's ability to interpret text and language. However, generally we are not conscious of this to be happening, or at times the process is so quick and subtle that we fail to, or hardly take, note of it. As such a conscious effort has to be made to be aware of that visualisation process or to induce the conscious portion of our brain to picturise the text.To understand what is implied by visualisation, consider the following news item.

The Supreme Court has noted with grave concern that lakhs of tonnes of food grain are rotting in government godowns. It has directed the government that food grain should not be allowed to rot and if need be it should be distributed free amongst the poor.

Did you notice any mental images being formed? If not then read again and try to take note of the images coming before you, or make a conscious effort to picturise what is being said.The word 'Supreme Court' may evolve the image of the court room or the Supreme Court building which we see on television or in pictures, or that of judges sitting in a court room. The words, 'grave concern' may induce images of some grim judges. The word, 'government', evokes different images in different persons. While rotting food grain may produce a picture of piles or sacks of grain lying in the open.

Though all words and ideas may not and cannot be converted into images, still the ability to visualise a reasonable portion of what is contained in the passage helps in the following ways:

(a) It helps in easier and better understanding of the letter and import of the text.

(b) It enables a clear and quicker recollection of the significant points when you are to read and answer the questions.

(c) It helps in remaining focussed on to the task of reading and understanding the passage. This is so, as our mind remains engaged in multiple activities, reading, understanding and visualising. Further, whenever the concentration wavers and the visual images stop being generated, the brain takes note of the concentration lapse and sets out for course correction.

2.5 Understanding the Import

Understanding the import implies, grasping what is the essence or the central theme of the passage and the underlying meaning that is being conveyed by its detailed contents. This is essential as the questions following the passage are least likely to be straight forward ones, where the correct choice can easily be picked by identifying the relevant portion of the passage from where the question has been taken. More often than not, the reader will have to analyse and apply his understanding of the substance of the passage to arrive at the right answer. As such while reading the passage, if need be, halt between lines or paragraphs and ask yourself, whether you have understood the logic

and the line of reasoning being adopted by the writer? Do not be in a haste to proceed reading further without fully comprehending the previous portion.

As mentioned earlier, the passages in the aptitude test are to assess the comprehension of the issue and not merely the letter. Therefore, the candidate will be required to apply his understanding to pick the correct choice of answer. This can only be done by understanding fully the text as well as the theme and essence of the passage. It is noteworthy that not only the passage as a whole, but its paragraphs and subparts also have themes and sub themes, which the author is trying to convey. It is essential to understand each part of the passage, its theme, essence of the entire passage and the line of argument adopted by the author. Consider the small news item quoted in Para 3.4. Though it consists of only two sentences it conveys many facts and ideas which are;

(a) Lakhs of tonnes of food grains are rotting in government godowns.

(b) Supreme Court has taken note of it and expressed serious concern.

(c) It has given two directions to the government:

(i) To prevent the rot

(ii) If required distribute it to the poor

The passages in the aptitude test will similarly contain many ideas, facts etc. which are knitted in a core theme of the passage. As a reader one has to understand what are the ideas and themes of the paragraphs and the core theme of the passage.

But why is this essential? Not merely to keep your mind occupied and prevent a concentration lapse. As mentioned earlier, though the question may spring from the passage itself, the answers are not going to be straight forward ones. They have to be discerned by applying the touchstone of your understanding.

Consider the following questions pertaining to the news item about the Supreme Court's direction.

(i) What do you consider as the most important reason for which the Supreme Court has given the aforesaid direction to the government?

(a) There is loss of valuable resource of the country, which is food grain.

(b) People are dying of hunger.

(c) Government has failed to discharge its duties properly.

Here the choice is not simple. The actual reason is apparently a combination of all of the above. As such if a choice (d) 'all of the above', were to be there, then the answer would have been obvious. But in absence of an obvious choice, to come to a conclusion one has to understand the theme of the passage.

The Supreme Court has expressed grave concern. This concern is not merely limited to the loss of resources and governments purported inefficiency, but more so at the loss of lives, which could be saved if the rotting food reaches the poor and hungry. Perhaps that is why it directed it to be distributed free amongst the poor, if need be. So, the most important reason should be choice (b).

2.6 Reading Speed

Reading speed is one aspect which is often thought to be an area of concern by some aspirants. They believe that a better reading speed would translate into their being able to solve the comprehension passage questions quicker. Therefore the faster they read the better it is. However this assumption is only partly true.

Reading faster would translate into being better, provided speed does not compromise the comprehension ability. If it does, then speed instead of being an advantage becomes a liability. After all marks are awarded for right answers and if comprehension of the passage and choice of correct answers is sacrificed, then quick reading may save you a few minutes but in turn may cause loss of valuable marks.

As such be wary of courses or techniques which offer you high 'speed reading' skills. Such skills may work on texts contained in newspapers, magazines or story books, where you would like to quickly skim through what has been written, but not on comprehension passages where the essence has to be understood and then the questions answered. Besides, unlike the categories mentioned above, the comprehension passages have high information and content density. Skipping or improper understanding of a portion or a key word, can impair your comprehension and the choice of the correct option.

But the converse of 'speed reading' is also not the correct option. Firstly, slow reading may not always mean the best comprehension of the subject. Secondly, the aptitude test paper will not give you luxury of all the time in world. Extra time taken on comprehension passages would mean lesser time availability for other sections. The implication is obvious, you need to read at an optimal speed, which is your fastest, without compromising your comprehension ability.

But then what is this optimal speed. Well, this may vary from person to person. But on an average a speed of 130-140 words a minute can be considered a good speed. So find out what is your own optimal speed and try to gradually improve upon it. This can be done with more practice and better concentration. But do always remember, it is the quality of comprehension which matters most.

HOW TO PREPARE: DEVELOPING SKILLS

Having known what skills and abilities are expected of you, the next logical step is to develop and hone them. Here are some suggestions and tips which you may find handy.

3.1 Improving Language Skills and Vocabulary

Proficiency in language and vocabulary, forms a fundamental aspect of comprehension. Improving on these skills pays you doubly. This is especially true, if 'English language and repertoire of words', has not been your forte or, if these skills got neglected in the past few years in your pursuit of professional studies. Candidates from the science background, engineering or medicine stream, at times do not get sufficient opportunities to work on these skills during the course of regular studies. As such it is advisable for them as well as for others, to brush up these skills prior to the Preliminary examination. Here is what can be done.

- **Read good books and articles** which have rich and fluent languages specially those on topical issues, which whet your thought process. Reading editorials of leading national dailies and articles of substance on concurrent and relevant issues can also be very helpful.
- **Note those words, whose meaning you are not aware of or sure about.** Check these up in a dictionary, specially the usage of words in different contexts. Jot then down in a diary or note book and revise it occasionally. This steadily but surely enriches your vocabulary.

3.2 Improve Comprehension Ability

Comprehension ability refers to a number of attributes which have been discussed earlier, such as understanding the literal text, theme and essence, ability to think on an issue etc. Here is what you can do to hone it.

- **Grasp the essence.** Whenever you read an article in a magazine, an editorial in a newspaper or a thought provoking essay, ask yourself what is the author trying to say? What is the main theme? What are the sub themes of different paragraphs (if any). This instills an ability to grasp the salient aspects of any write up.
- **Visualise while you read.** As discussed in the previous section, try to picturise what the author is trying to convey. This also gives a boost to your skill of mental imagery and ability to co-relate words, with thoughts and ideas.
- **Reflect and ponder over what you have read.** Merely passive reading is not sufficient. Ask yourself whether or not you are in agreement with what the author has to say. Is the view expressed balanced? Such an exercise sharpens your analytical abilities and understanding of the issues.
- **Practise and Practise more.** Please remember there is no substitute for actual practice. No matter how much of theory you have mastered, it is meaningless unless you have tested your abilities on the touchstone of actual practice. Practice reveals your shortcomings and helps you to overcome them. It indicates where do you stand and with time and learning, whether you have shown improvement or not. It helps you understand what are your areas of strength and weakness and how to plan your strategy.

A number of exercise sets have been given to give you good practice. While attempting the first few exercise sets, your objective should be to get in the grove and to identify the areas of comprehension ability (if any), you need to work upon (viz. vocabulary, understanding the import, concentration, identifying the correct choice etc.). Do not be in a hurry to complete all exercises within a few days. Try to work upon your skills and then proceed to the next set. Progress to the final exercise set when you feel you have got a hang of the test of comprehension. At this stage it will be prudent to time your practice sessions so that you are aware how much time you are likely to take in attempting a comprehension passage and could plan your strategy of attempting the paper accordingly.

HOW TO ANSWER

Here are some suggestions and tips which will be helpful while answering, in ascertaining the right option, saving time and avoiding mistakes.

- While going through the passage try to **underline the relevant words, portions and key concepts** which you consider the passage is trying to convey. This not only helps in better understanding of the passage but helps you in quickly locating the relevant portions, if you need to refer to, while answering. Besides, it is possible that some questions may emanate from them.
- **Read the questions and options properly.** Once you have read the question, pause for a second and repeat the question to yourself, or ask yourself what have you understood. This helps in clearing any misconceptions bred by your own haste. Do not hesitate to read the question again or to go back to the relevant portion in the passage to recollect, if need be.

- Look out for expressions such as, 'which is not the correct reasoning' (a question in negative) or, 'which is not the suggestion made by the author'. In haste one is likely to miss the word 'not' and select the wrong choice.

- In certain cases the question asks, "what as per the author/passage are the reasons....?". Here the question has to be answered strictly as per views expressed/facts stated in the passage. Such questions should not be answered on the basis of a commonly known fact which may otherwise even be true.

- **Read all the options carefully before deciding your answer.** Do not jump to a conclusion just after reading the first or the second choice. The answer choices offered may be quite similar to each other with only a fine difference between them. As such an incorrect choice may also appear to be the right answer, until you read the subsequent choices and realise that one of them appears to be a better option.

- Many a time the choices of answers are such, that all or more than one of them, may be partly correct. The fully correct one or the right option in such cases can only be picked up by very careful reading and understanding the theme and the essence of the passage.

- In some questions, one or more than one of the options may be totally illogical or patently incorrect. Here **the technique of 'selection' by elimination may be applied.** After the first reading of the options, strike out the ones which are totally incorrect. Choosing amongst the remaining options, becomes easier. Besides, this saves some time in arriving at a conclusion, as one does not have to go through it again over the struck out options.

- **Pre-judging or pre-phrasing** your reply is also a technique which works well in some cases. If the question is such that you can articulate/frame your reply without having to look at the given options, then it is called pre-judging or pre-phrasing the reply. Once you have pre-judged the reply and you do find an option closely matching your reply, then the probability of it being the correct choice becomes quite high. This helps in saving time as you are able to conclude which is the right option rather quickly.

- **Budgeting your time for various sections** and chalking out before hand strategy is always very helpful. This means how much time do you propose to allot to each section and to each question in that section should be planned before hand. If you are stuck at a question or have already over shot the time but cannot figure out the correct answer, then it is always prudent to leave the question and not waste further time on it. You may revisit the question later (if time permits) or make an intelligent guess between the possible choices. However, guess work is ill advised where your confidence is low about the correctness of the choice, as negative marks will be awarded for wrong answers.

For the comprehension passages which may be having 10 marks (4 questions), budgeting around six to seven minutes for reading the entire passage and answering the questions is considered reasonable. The aptitude test paper will be of 200 marks and 120 minutes will be the time permitted. As such if one allots a minute for every one and a half marks, then in two hours time one would be able to attempt questions worth 180 marks. This should be considered a good attempt. Thus, for a comprehension passage of 10 marks, keeping roughly six to seven minutes budgeted, is a reasonable time allocation. But if you are taking much more than this time span, then either be very confident of your answers (so that you get some return for the, heavy investments made in terms of time) or move to another question or section where you think you can fare better.

LOGICAL COMPREHENSION

In the General Studies Paper 2 of 2015, a distinct category of comprehension passages were introduced. A number of small passages, with a singular question emanating from them, were incorporated in the question paper. The passages and questions had certain distinct features as compared to the regular comprehension passages, which are:

(a) The passages were quite brief ranging from 30 words to around 150 words, as compared to the regular passages of 200–500 words.

(b) Each such passage had only one question emanating from it as compared to 2-6 questions in the regular passages.

(c) The nature of questions was such that it required not only the understanding of the passage but a logical appreciation of the import of the passage as well. This would be tested with the candidates being required to select/ascertain the following;

- *What best sums up the passage*
- *The most logical corollary to what is stated in the passage*
- *Most logical rational and crucial message of the passage*

- *Convincing explanation to what is stated in the passage*
- *What argument/comment can best sum up the author's view point*
- *Most rational and critical inference of the passage*
- *Which statement supports most/is least essential as a part of argument*
- *Which is the most valid assumption which would support what is stated in the passage*

The **traits essential to solve such comprehension passages primarily remain the same** as that required for the regular passages. Albeit the time taken to go through is less as the passages are small. However, **candidates must go through the passages very carefully** and not skim through them. **Effort should be made to understand the import of what is stated in each and every sentence.**

More often than not, the questions will be such that a direct answer cannot be found anywhere in the passage. As has been highlighted earlier, once the passage has been understood and underlying logic has been grasped, then the questions will test its understanding and application.

Some such passages are included in the exercise set as well as in the model question papers which will enable the readers to get a hang of such logical comprehension.

WHAT ARE SENTENCE COMPLETION QUESTIONS?

Sentence completion questions test your vocabulary skills as well as your reading ability. These problems contain a single sentence expressing a complete idea that can be understood without any additional information. This is quite unlike the reading comprehension questions, which require you to read long passages. Each sentence contains one, two or three blanks, which need to be filled up appropriately. These questions typically contain four options to fill in the blanks in the sentence. From these choices, you need to select the words or phrases that fit into the blanks to best complete the sentence.

This question type tests the student's ability to understand the main idea of the sentence and the logical structure of the sentence. It also tests the ability of the student to anticipate what idea conveyed by a particular word will most aptly fit into the blank provided. Besides, your vocabulary is also tested because there is not much you can do if you are unaware of the word/s contained in the question or in the options. Your knowledge of roots, prefixes and suffixes will come in handy.

In order to successfully solve the sentence completion section the student should have a strong understanding of perceiving the relationships within the sentence. These relationships might include the use of equivalents, analogies, parallel sets, contrasts and word clusters. Solving the sentence completion section will draw on your adeptness and facility with antonyms and synonyms, your understanding of parallel sets, and the breadth and depth of your general vocabulary.

Note for the Students:

Needless to say, having a good vocabulary helps in doing better at this question type. The student is hence advised to work on his/her vocabulary simultaneously in order to improve his/her ability at this type of question. However, even if you are caught short in your vocabulary by a particular question you can still try to solve it by following a few rules which are enumerated below.

CHAPTER 2

Critical Reasoning and Paragraph Comprehension

INTRODUCTION

The first thing you need to realise as you start this chapter, is that **critical reasoning is strategically important.** Over the past decade these questions have been consistently present in a good number in various aptitude tests.

Your ability to solve this question type depends on your ability to recognise and evaluate argumentative logic. The better you are at understanding arguments, the better you will be at questions related to critical reasoning.

Argumentation being a common fabric of our day to day life, doing well at Critical Reasoning (CR) questions depends more on your ability to reason out logically, than on your specific knowledge of a language.

What is Critical Reasoning?

The typical structure of CR questions is that of a short passage (mostly consisting of a single paragraph) followed by a question on the basis of the paragraph.

The typical CR passage is anything between 50 words to 200 words long and necessarily contains an **argument**. As mentioned earlier an argument will always have a **claim** supported by **reasons/evidences**.

Characteristics of a Critical Reasoning Passage

While writing an argument the author's principal objective is to persuade the reader about his/her point of view. A successfully presented argument convinces the reader about the author's point of view. In order to do so, not only does the author have to present his point of view, but he also needs to support it with reasons—after all the reader needs to know why he should agree with the author's point of view. Hence, the author has to provide enough support for his point of view in order to convince the reader. This support may be in the form of reasons or evidences. **Note:** The points that **support** an argument are also called as the **premises** of the argument.

For each author's work that you read, you must be able to do two things:

1. identify the specific position the author takes on a particular issue, and
2. identify the ways in which the author provides support for that position.

If you can identify 1) *the specific position* and 2) *all the statements that support it*, then you have identified what is called the argument of that essay.

One quick way to remember an argument is:

argument = specific position + supporting points

In English composition, an argument is usually discussed in the following way:

argument = main claim + supporting evidence

In critical reasoning passages, an argument is usually discussed in this way:

argument = conclusion + reasons

In all three versions, argument *means essentially the same thing.* This leads us to the definition of an argument.

The Structure of Every Argument is:

Because of **these** (reasons or facts) we should conclude **this** (claim/conclusion). The illustration given alongside will make this clear to you.

An argument is composed of two kinds of statements:

(1) the conclusion (main claim) is that statement which follows from the other statements, and

(2) the reasons (evidence) are those statements which are intended to support the conclusion.

Support of an Argument

Consider the following passage that appeared in a newspaper editorial. The passage below has a claim supported by reasons.

Flexitime, or flexibility of working hours, has become popular amongst corporates in recent times. (This is the claim). Many corporates have found that flextime has several advantages. The most obvious advantage is less absenteeism. When employees can choose working hours that meet their needs, they are less likely to take time off. Another advantage of flextime is more efficient use of the business office. The additional hours that a company is "open for business" often converts into higher productivity and greater profits.

Besides, giving employees a choice of their working hours allows them to exert more control over their working environment. This leads to increased job satisfaction and less employee turnover.

Similarly there could be a passage having a claim supported by facts. (Please recollect that 'facts' are statements/information that can be physically verified. They might prove to be true or false on verification.)

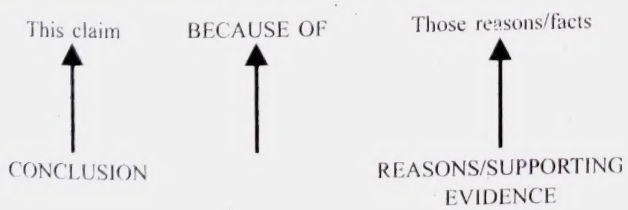
Consider the following passage:

Country X's relations with Country Y have varied from being stormy and incordial at times to being icy cold and impersonal at other times. There have been periods in the histories of the two countries when their relationships have been dictated by the military maneuvers around their borders—for instance the times preceding 1945, 1957 and the Kampaundril wars—while at other times the relationship has been cold and impersonal at best—due to Country X's constant refrain that Country Y has been instigating terrorism within its borders.

Let us now look at some additional points that you need to keep in mind while reading for the conclusion and the supports in a paragraph.

Recognising the Structure of an Argument

Now that we have a more definite sense of what an argument is, let's explore its structure further. It's important to emphasise that every argument has the same structure.



Let us look at a few examples of the structure of an argument.

EXAMPLE 1 Abortion should be allowed **BECAUSE** a woman has the right to decide what happens in and to her own body.
This [Abortion should be allowed]

BECAUSE OF

That [a woman has the right to decide what happens in and to her own body]

EXAMPLE 2 Smoking should be banned **BECAUSE** it is harmful to everyone's health.
This [Smoking should be banned]

BECAUSE OF

That [it is harmful to everyone's health]

What's important about argument structure (this—because of-that) is that we can use it as a test to help us decide which statement is the conclusion and which statement is the reason. It helps us to identify the parts of the argument. But it often happens that the **"BECAUSE"** is not stated, and we have to decide where it goes. For instance, take the following example:

EXAMPLE

(a) Whatever Sue says about values is wrong.

(b) After all, she's an unabashed alcoholic.

Let's identify the this-because of-that structure in this argument.

Think of the "this" and the "that" as two empty slots in the following way:

__this_____ BECAUSE OF_that_____

If you're not sure which statement is the conclusion and which one is the reason, try putting (b) in the "this" slot and (a) in the "that" slot like this:

(b) *After all, she's an unabashed alcoholic* **BECAUSE**

(a) *whatever she says about values is wrong.*

Does this make sense? **No, it doesn't.**

Sentence (b) does not follow from sentence (a);

Sentence (a) does not support sentence (b).

Let's try it the other way:

(a) *Whatever she says about values is wrong* **BECAUSE**

(b) *after all, she's an unabashed alcoholic.*

This makes sense. Of course, this *doesn't* mean that it's a good argument. It merely shows *which part is the conclusion* and *which part is the reason.* Clearly, sentence (a) is supposed to follow from sentence (b) and sentence (b) is supposed to provide support, evidence or proof for sentence (a).

Conclusions, Reasons, Assumptions and Inferences

Strategies for Spotting Conclusions

1. Know where to look for conclusions in certain parts of the essay (introductory paragraph and concluding paragraph).

2. Look for indicator words which signal conclusions. Words such as *therefore, thus, hence, so, in conclusion, as a result, in short* and *the point is* indicate conclusions.

Consider the following example.

"Nuclear power plants are hazardous to human life and the environment. Thus, these plants should no longer be built."

The *indicator word* 'thus' at the beginning of the second sentence indicates the conclusion: these plants should no longer be built.

3. If no conclusion or main claim is stated, you can construct it by finding the main question which the essay directly addresses.

Strategies for Spotting Reasons

1. Know where to look for reasons in certain parts of the essay (in the body of the essay). Remember that reasons can include fats, surveys, statistics, value judgements and general opinions.

2. Look for indicator words which signal reasons, words such as *because, since, on account of, for, in view of the fact that,* and *for the reason that.* These words function in the same way as the word *because.*

For example in the following argument:

"On account of his not finishing the sixth grade and in view of the fact that he wears glasses, I can't believe Jim's account of what happened that night."

The phrase on account of indicates the first reason (Jim didn't finish the sixth grade) and the phrase in view of the fact that indicates the second reason (Jim wears glasses). In standard argument form, the argument looks like this:

Conclusion: I can't believe Jim's account of what happened that night.

Reasons: 1. Jim didn't finish the sixth grade.

2. Jim wears glasses.

Please remember that we are not evaluating the argument at this point; we are trying only to reproduce it faithfully. (But your hunch is right; this is a very poor argument!)

3. After you have found the conclusion, ask the question WHY? Any sentence in the essay that responds directly to this question is a reason. In the above example, WHY does the author not believe Jim's account of what happened? BECAUSE Jim didn't finish the sixth grade and BECAUSE Jim wears glasses.

Question Type 1: The Strengthening of an Argument

The claim of an argument is supported by reasons or evidences. In questions on strengthening the argument, the question asks you to select from amongst the four/five options, the one that strengthens the argument. Arguments can be strengthened in two ways: either through the introduction of some supporting evidence or the introduction of some supporting reasons. Hence, these questions might ask you to select an option that provides either supporting evidence or supporting reasons.

Besides if we have an option that strengthens an assumption that is the key to the argument's claim, then that option will also strengthen the argument.

In a figurative sense, if you look at the argument as a building with supports, then supporting evidences/reasons provide us with additional support to the claim of the question. In such questions, while evaluating the options, you should try to assess which option best supports the claim of the argument. The strengthening evidence/reason might be stronger/equal to/or weaker than the explicitly stated evidences/reasons in the argument. However, in order to find out the correct answer to such questions you do not need to compare the quality of the support an option provides with respect to the explicitly stated supports. All you need to do is compare the respective

options and try to see which option best supports the claim of the argument. By evaluating the relative strength of the support provided to the claim of the argument you can easily identify the one that best supports the claim.

The following question will make this question type clear to you.

One of the most important and constructive reforms in national politics is the abolition of the post of the State Ministers in the various departments.

Each of the following, if true, would strengthen the above argument, except

(a) There are few, if any, specific duties or responsibilities assigned to the State Minister in any department.

(b) A historian claimed that the post was "superfluous".

(c) People of Cabinet Minister caliber normally refuse the post if offered a ministership in the guise of a State Minister.

(d) The office is used as a means of appeasing regional parties, by giving their MPs ministerial status and perks without giving them any significant responsibilities.

The correct answer is (b).

Question Type 2: The Weakening of an Argument

These questions are very similar to strengthening argument questions—the only difference being that they are on the other side of the fence. Similar to the strengthening of arguments, weakening of arguments can also be done by the introduction of additional evidence and/or reasons that attack the basis of the claim of the argument.

After identifying the claim in an argument and the various supports provided for the claim, you will need to evaluate each option for the degree to which it goes towards weakening the argument. The evidence/reason that most/least seriously weakens the supports of the argument's claim would be the answer (depending upon what you have been asked to identify).

In a figurative sense, the weakening evidence/reasons are like attacks on the pillars of the building, i.e., they attack the supports of the argument's claim. Your judgment needs to tell you how serious the attack is.

Besides if we have an option that weakens an assumption that is the key to the argument's claim, then that option will also have the effect of weakening the argument.

Let us look at a few examples that will make this question type clear to you.

EXAMPLE 1

Before the arrival of a new trainer, the sales output in AMS Learning Systems Ltd had been rising by 20% per year on the average over the past ten years. However, after new training innovations by the trainer (which included computerisation of training processes and reductions in the need for additional work force) annual sales output has only risen by 10% this year. It appears that Joe's innovations have caused the reduction in the annual growth rate.

Which of the following, if true, would most seriously weaken the conclusion above?

(a) The investment in computerisation has a provision for depreciation of the cost of the computers.

(b) Increases in selling price did not follow increases in the cost of the inputs.

(c) The innovations brought in by the new trainer were intended as long-term investment and not made for short-term profit growth.

(d) General demand for the training provided by the company has declined.

The correct answer is (d).

EXAMPLE 2

In the past, to run for one's country in the Asiad was the ultimate achievement of any athlete. Nowadays, an athlete's motives are more and more influenced by financial gain, and consequently we do not see our best athletes in the Asiad, which is still only for amateurs.

Which of the following will most weaken the above conclusion?

(a) The publicity and fame that can be achieved by competing in the Asiad makes the athletes who do so more "marketable" by agents and potential sponsors. Thus they can earn a lot of money even while retaining their amateur status.

(b) The spirit of the Asiad places emphasis on participation rather than on the winning of the race.

(c) A leading columnist recently argued on the basis of concrete evidence that our best Asiad athletes already receive enough in terms of promotions and sponsorships.

(d) It has been suggested that professional athletes should be allowed to compete in the games.

The correct answer is (a).

EXAMPLE 3

In these times of growing economic turbulence, unless new reserves are found soon, the world's supply of natural gases is being depleted in such a way that with demand continuing to grow at the present rates, reserves will be exhausted by the year 2200 AD.

Which of the following, if true, will most weaken the above argument?

(a) There has been a slowdown in the rate of increase in world demand for natural gases over the last decade from 20% to 10%.

(b) It has been known for many years that there are vast stocks of natural gases under Antarctica which have yet to be economically exploited.

(c) Electricity is being used increasingly in place of natural gases for many industrial and domestic uses.

(d) None of the above.

The correct answer is (d).

EXAMPLE 4

In accordance with their powers, many zilla panchayats are introducing chlorination of the drinking water provided to families through the water supply system. This follows the conclusion of 10 years of research that the process ensures that children and adults receive the required intake of fluoride that will strengthen teeth. The maximum level has been set at one part per million. However, there are many who object, claiming that chlorination removes freedom of choice.

Which of the following will weaken the claim of the proponents of chlorination?

(a) Chlorination over a certain prescribed level has been shown to lead to a general weakening of teeth.

(b) There is no record of the long-term effects of drinking chlorinated water on dental and general health.

(c) In a study done at the grassroots level, it was found that some people who believe to be affected by chlorination claim that they have not had sufficient opportunity to voice their views about the issue.

(d) Water already contains natural chlorine.

The correct answer is (b).

PRACTICE EXERCISES

Exercise-1

1. Monopoly is characterised by an absence of competition. The Singhania Company realises that its operations are in competitive industries.

   Which of the following conclusions may be inferred from the above?

   (a) The Singhania Company's market is not monopolistic.

   (b) Monopoly is defined as one seller in a market.

   (c) The Singhania Company has no domestic competitors.

   (d) The Singhania Company is publicly owned.

2. People in the South have observed that heavy frost is usually preceded by a full moon. They are convinced that the full moon somehow generates the frost.

   Which of the following, if true, would weaken the people's conviction?

   (a) The temperature must fall below 10 degrees Celsius (50 degrees Fahrenheit) for frost to occur.

   (b) Absence of a cloud cover cools the ground which causes frost.

   (c) People are superstitious.

   (d) People are not experts in meteorology.

3. Professor Jha told his class that the method of student evaluation of teachers is not a valid measure of teaching quality. Students should fill out questionnaires at the end of the semester when courses have been completed.

   Which of the following, if true, provides support for Professor Jha's proposal?

CHAPTER 1

Introduction to Logical Reasoning

Ever since the start of time, man has always been surrounded by logic (time and logic seem to have coexisted) —it exists all around us, and is all encompassing. There is implicit logic in all human and natural activities—right from the primary level of logic seen in our day-to-day lives to its very advanced form which operates the machines & tools we use for our day-to-day work. Every subject we study, every product we build, every activity we undertake is guided by its own inherent logic.

For instance, when faced with the situation of starting a car, switching on an electric appliance like an electric bulb, etc., we use logic inherently. In fact, it would be difficult to imagine life today without even a thought on most of the logical structures that we use inherently.

LOGIC AND LANGUAGE

The ancient Greeks were the first to study logic as a subject in depth. The lack of any systematic notation for the process of logic during its initial development led the Greeks to rely extensively on the use of language to explain logic. Each one of us even today instinctively uses language to explain what we are doing. Thus we use logical language in each of the following cases—

1. Where is my key?

Ans: It's on the blue table.

Or

It's in the second shelf from the bottom inside your cupboard.

2. Could you direct me to Mr Mehta's house?

Ans: Proceed straight from here and take the right turn from the second crossing. Move about 100 metres after the right turn and you will reach a 'Y' junction. Take the left leg of the Y and this road leads to a dead end. Mr Mehta lives on the second floor of the second last house on the left of this road.

3. If I put water in a working refrigerator it will become cold.
4. To turn on the car, one needs to switch on the ignition.

Thus, each of us comes across millions of such everyday situations where we use logic inherently as part of our day-to-day language.

The study of logic by the Greeks was largely confined to the study and documentation of logical language. However, the problems of understanding logic through language are very intricate, since this approach becomes extremely complex and unwieldy, the moment the logical string becomes longer. This complexity was the reason that when Aristotle summed up Greek logic in his *Treatise on Logic* in the 4th century BC, many of the greatest minds were at a loss to understand it.

Symbolic Logic

It was only in the late 19th century when Gottlob Frege brought about a revolution in the whole field of reasoning by inventing 'symbolic logic'— the use of symbols to represent ideas; that the next phase of development in logical thought started. With this improvement of notation, logical and mathematical ideas could be precisely written down perhaps for the first time. The inconsistencies and vagueness of language were overcome through the use of symbols to denote logical thoughts. The development of 'symbolic logic' further led to the development of 'logical thinking'.

Consider the statement: "If a car has poor air conditioning or low fuel efficiency then it is not a nice car, then the fact that a car is nice means that it will have neither poor air conditioning nor low fuel efficiency".

This long language string used to express the above logic can be condensed using the following symbols:

P (poor air conditioning) & L (low fuel efficiency) & N(C) (Nice car).

Thus, if P or L then not N(C) then N(C) means not P or L.

The words OR, NOT & AND all can have their own logical symbols.

Thus OR is 'v', NOT is ~ AND is ^.

Thus, the above sentence can be condensed in its logical form as:

$$(P \vee L) \rightarrow (\sim N(C)) \rightarrow N(C) \rightarrow (\sim P) \& (\sim L).$$

In fact, to reduce ambiguity, tediousness and complexity of language based interpretation, solving logical problems involves as its first step the interpretation of the language of the question and conversion of the language of the question into symbolic form. Moreover, this also aids in cutting down the time required to solve complex reasoning questions.

Thus, when the language of logic is converted to symbolic form, the truth of such a sentence depends only on its logical form and not on its content.

Any similar sentence (using totally different language), which uses the same logical form will be true under the same circumstances. The process of mastering reasoning then has to take as its first step the understanding of the process of conversion of language into symbolic logic. In fact, the process of solving logical problems is greatly eased through the use of symbols to document the language.

Some Standard Symbols that You can Use for Logical Language

NOT	~
OR	v
AND	+
If and Only if	≡
Not, if	≠
If, Then	⊃

Besides, you would do well keeping in mind the following factors while creating a symbolic framework for your question:

- → Proper nouns should always be denoted by capitals.
- → In questions where the sex of the proper noun is crucial to solve the questions, you can denote the female by underlining the capital letter used.
- → As you start solving Reasoning Questions, create your own set of symbols for standard relational sentence structures. *(See the highlighted box)*:

(1) Is next to
Thus, A is next to B can be: AB v BA

(2) (a) Is to the immediate left of
Thus, A is to the immediate left of B can be: AB

(b) Is to the left of
Thus, A is to the left of B can be: A___B

(3) (a) Are at the ends
Thus A and B are at the ends means:
(A _____ B) v (B _____ A)

(b) Is at the extreme left
Thus A is at the extreme left means:
(A_____)

(4) Is more than/taller than/Greater than
Thus A is greater than/ taller than/ more than B and C can be symbolically represented by:
A > (B + C) [Note here that we do not know the relationship between B and C] or A > (B, C)

(5) Is Less than/Shorter than/lower than
Thus A is less than/shorter than/lower than B and C can be symbolically represented by:
A < (B + C) [Again, note here that we do not know the relationship between B and C] or A < (B, C)

(6) Is between
Thus, A is between B and C means that A can be anywhere between B and C.
(B___A___C) v (C___A___B)

(7) Is the son of
Thus, A is the son of $\rightarrow \frac{B}{A}$

(8) Is the daughter of
Thus, A is the daughter of B $\rightarrow \frac{B}{A}$

(9) Is the parent of
Thus, A is the parent of B $\rightarrow \frac{A}{B}$ V $\frac{A}{B}$

(10) Is the spouse of
Thus, A is the spouse of (B → A—B v A—B)

(11) Is the wife of
Thus, A is the wife of B → A—B

(12) Is the sibling of
Thus, A is the sibling of B → (A..B) v (A .. B) v (A..B) v (A..B)

Some Important Logical Manoeuvers:

(1) If, Then
The condition If A, then B leads to the following valid conclusions
Valid Reasoning 1: A therefore B.
Valid Reasoning 2: Not B, therefore not A.
At the same time If A then B also throws up the following invalid conclusions.
Invalid Reasoning 1: B, therefore A.
Invalid Reasoning 2: Not A, therefore not B.
The above structures of logical thought can be illustrated through the following examples:
If A teaches, B will go to the movies.
Valid Reasoning 1: A teaches, therefore B will go to the movies.
Valid Reasoning 2: B has not gone to the movies, therefore A did not teach.
Invalid Reasoning 1: B went to the movies, therefore A must have taught. This is an invalid line of reasoning.
Invalid Reasoning 2: A did not teach, therefore B did not go to the movies. This too is invalid.

(2) If and Only If
The condition If and Only If A, then B leads to the following valid conclusions.
Valid Reasoning 1: A, therefore B.
Valid Reasoning 2: Not B, therefore not A.

Valid Reasoning 3: B therefore A.
Valid Reasoning 4: Not A therefore not B.
The above structures of logical thought can be illustrated through the following examples:
If and only if A teaches, B will go to the movies.
Valid Reasoning 1: A teaches, therefore B will go to the movies.
Valid Reasoning 2: B has not gone to the movies, therefore A did not teach.
Valid Reasoning 3: B went to the movies, therefore A must have taught.
Valid Reasoning 4: A did not teach, therefore B did not go to the movies.

(3) Either Or
Either A or B
Valid Reasoning 1: Not A then B
Valid Reasoning 2: Not B then A
Valid Reasoning 3: A then Not B
Valid Reasoning 4: B then Not A
The above structures of logical thought can be illustrated through the following examples:
Either A teaches or B goes to the movies.
Valid Reasoning 1: A does not teach, therefore B will go to the movies.
Valid Reasoning 2: B has not gone to the movies, therefore A must have taught.
Valid Reasoning 3: A taught, then B did not go to the movies.
Valid Reasoning 4: B went to the movies, then A did not teach.

(4) If, Then Not
If A then Not B:
Valid Reasoning 1: A then not B
Invalid Reasoning 1: Not B then A
Valid Reasoning 2: B then Not A
The above structures of logical thought can be illustrated through the following examples:
If A teaches, then B will not go to the movies.
Valid Reasoning 1: A teaches, therefore B will not go to the movies.
Invalid Reasoning 1: B has not gone to the movies, therefore A taught.
Valid Reasoning 2: B went to the movies, therefore A did not teach.

As a student it is important for you to follow a few standard steps while solving logical questions:

1. Take a complete 'preview of the situation' to clearly understand the context. Remember to accept the situation as it is given.
2. Read each and every part of the question carefully. You should concentrate hard and focus fully while reading the question. This is a very important prerequisite for solving questions on logic since very often in long sentences there will be individual single words which will transform the meaning of the sentences. If you fail to take into account these words, the end result will be errors in logic & deduction.

Let us now proceed to understand how all this applies to real life problem solving through examining questions which have been asked in different competitive exams and CAT.

EXAMPLE 1 A party was held at the house of the Mehtas. There were five other couples present (besides Mr and Mrs Mehta), and many, but not all, pairs of people shook hands. Nobody shook hands with anyone twice, and nobody shook hands with his/her spouse. Both the host and hostess shook some hands.

At the end of the party, Mr Mehta polled each person present to see how many hands each person (other than himself) shook. Each person gave a different answer. Determine how many hands Mrs Mehta must have shaken.

Can we prove that it was not Mrs Mehta who shook 10 hands?

SOLUTION

Let there be 5 couples:

A — *A*
B — *B*
C — *C*
D — *D*
E — *E*
& M — *M*

Deduction 1 From the condition that nobody shook hands with his/her spouse, it is clear that none of the twelve people in the party shook more than 10 hands.

(Since, nobody shakes hands with himself or his/her spouse, it leaves a maximum of 10 people to shake hands with).

Deduction 2 Mr Mehta has asked the question to eleven different people and each of them has given a different answer. Also, the highest answer anyone could have possibly given is 10. Hence, the only way to distribute different numbers of hand shakes amongst the 11 people is:

0,1,2,3,4,5,6,7,8,9,10. [*Note*: somebody shook 0 hands and somebody shook 10].

Deduction 3 Since the host & hostess have both shaken some hands, the person who shook '0' hands cannot be either M or $\underline{M}$. It has to be one of the other 10 people in the party.

[At this point you need to realise that in the context of this problem A, $\underline{A}$, B, $\underline{B}$, C, $\underline{C}$, D, $\underline{D}$, E & $\underline{E}$ are alike, i.e.

there is no logical difference amongst these 10 and you have exactly the same information about each of these 10 people. However, Mrs Mehta is different because she stands out as the hostess as well as the wife of the person who has asked the question].

Since all ten guests are the same, assume 'A' shook no hands. This leads us to the following deduction.

Deduction 4 Take any one person apart from A & A; say B. B will not shake hands with himself & his wife. Besides, B will also not shake hands with A (who has shaken no hands). Thus, B can shake a maximum of 9 hands and will thus not be the person to shake 10 hands.

What applies to B, applies to B, C, C, D, D, E, E and M.

Hence, A is the only person who could have shaken 10 hands. Hence, amongst the couple A & A, if we suppose that A had shaken 0 hands, then A must have shaken 10 hands.

Note: The main result here is that, out of the people to whom M has asked the question, and amongst whom we have to distribute the numbers 0 to 10, there has to be a couple who has had 0 & 10 handshakes. It could be any of the five couples, but it cannot be M who has either 0 or 10 handshakes.

We now proceed, using the same line of reason as follows.

Deduction 5 Suppose B has 1 handshake — he must have shaken hands with A (who has shaken everybody's hands she can).

Then, B wouldn't have shaken hands with anyone out of A, B, C, C, D, D, E, E & M. At this point the following picture emerges:

A — 0	A — 10
B — 1	B —
C —	C —
D —	D —
E	E
M	

Numbers left to be allocated — 2, 3, 4, 5, 6, 7, 8, 9.

Considering C, as a general case, he cannot shake hands with C, A (who shook no hands) & B (who shook hands only with A. This is mandatory since A has shaken hands with 10 people).

Thus, C can shake hands with a maximum of 8 people and this deduction will be true for C, D, D, E, E & M too. Hence, the only person who could get 9 handshakes is B.

Thus, we conclude that just like 0 and 10 handshakes were in one pair, similarly 1 and 9 handshakes too have to be part of one pair of husband and wife.

Similar deductions will lead to the realisation that 2 & 8, 3 & 7 and 4 & 6 handshakes will also occur for couples amongst the 11 people questioned.

Hence, M must have shaken 5 hands.

Let us consider another example:

EXAMPLE 2 Consider the following grid:

L		P
M	O	Q
N		R

Each letter in the above grid represents a different digit from 0 to 9, such that

$L \times M \times N = M \times O \times Q = P \times Q \times R$. Find the value of 'O'.

SOLUTION

In order to solve such a question, one needs to proceed systematically making one deduction at a time.

Deduction 1 There are 7 alphabets and 10 digits. We need to somehow eliminate 3 of these 10 to define the 7 digits required to be allocated.

It is obvious that '0' cannot be used because if we make any alphabet '0' we will end up with a product that is '0' in one or a maximum of two of the three cases.

Deduction 2 You need to find a product which can be made in three different ways.

Deduction 3 Two of these three ways have to be independent of each other with no matching digits and the third way has to be drawn out of one digit each from the first two ways and one independent digit that has not been used.

Also, at this point there are 9 digits and 2 more to be eliminated.

Now we move to the process of Trial & Error.

Notes on Interpretation

Trial & Error is one of the most useful processes for solving questions based on reasoning. Principally there are three ways for carrying out trial & error search.

(1) Complete trial & error.
(2) Directed trial & error.
(3) Blind trial & error.

The blind trial and error is what most students practice and hence, are unable to solve logical questions. Since they do not use their deductive logic to do a more focused search, they end up going round in circles while trying to solve such questions.

Instead, directed trial & error and comprehensive trial & error are superior problem solving processes and therefore, score above the complete trial and error method.

Application of Directed Trial & Error: The question above is a classical situation warranting a directed trial & error. Hence, this is best illustrated through the example.

At this stage, we are at a situation where we know:

1, 2, 3, 4, 5, 6, 7, 8 and 9 have to be allocated to L, M, N, O, P, Q and R.

At this point, take a call as to whether you want to use 9 or not? 9 and 1 are different from the other numbers primarily because they are the highest and lowest numbers respectively and also because 9 gives us "maximum room for manoeuvering" the question as compared to the other numbers, while 1 is a useful tool for a third multiplier if we do not want to change the value of the multiplication.

Note: The student should analogise the thinking process applied here to the thinking process used to unravel a ball of wool which has got entangled.

To disentangle an entangled ball of wool, we need to search the end of the ball. Once you identify either end, the remaining process of disentangling the ball requires very elementary logic coupled with patience and perseverance. The logic for picking up the end point of the ball of wool is that it is different from all other points in the ball.

Similarly, in reasoning questions, we need to identify things/objects/people which are different from other things/objects/people and start our solution from there.

After that, the whole process becomes one of the use of elementary logic for elementary deductions coupled with patience and perseverance.

In the question under consideration, if we take a call on using '9' and decide to do so, we can then deduce that the required product has to be a multiple of 9.

[Remember that at this point of time, we have ignored the line of thinking that neglects 9. We will have to consider it, if we do not get an answer by using 9].

Since the product has to be a multiple of 9, assume that the product is 36. But this product eliminates the use of 8,7 & 5 and leaves only 6 digits. Going below 36 as a product for further trial and error will further reduce the number of possibilities. Hence, let us try to go to the higher extreme & try to experiment with the number 72.

We see that $72 = 3^2 \times 2^3$ and can be formed by $9 \times 8 \times 1$ or $6 \times 3 \times 4$ or $9 \times 4 \times 2$.

This satisfies our deduction that 72 gives three ways of solving the question. Also, the second requirement that there should be two ways which are independent of each other and a third way which uses one term each from the two independent ways and one unique term is also satisfied.

Hence, $6 \times 3 \times 4$ is independent of $9 \times 8 \times 1$ in all its digits. Also, $9 \times 4 \times 2$ uses the number 9 from $9 \times 8 \times 1$ & 4 from $6 \times 3 \times 4$.

Hence, the following possibility emerges:

L = 6	P = 8	
M = 4	O = 2	Q = 9
N = 3	R = 1	

You need to understand here that, the digit '2' is the only one which is fixed to its place in the grid. All the other digits can be changed. Thus, we can have alternative arrangements like:

L = 3		P = 1
M = 4	O = 2	Q = 9
N = 6		R = 8
OR		
L = 8		P = 6
M = 9	O = 2	Q = 4
N = 1		R = 3

The only thing that is fixed is the number 2 for O.

EXAMPLE 3 Let us now consider another question, which is a classic case of complete or comprehensive trial & error method.

Two people A & B are playing a game. Both A & B are logical people. There are two boxes on the table. One of them contains 9 balls, the other contains 4 balls. In this game, the players are supposed to take alternate turns of picking up balls according to the following rules:

(a) Pick up as many balls as you want to from any box.
(b) Pick up an equal number of balls from both boxes. (if you pick from both boxes).

The person who picks up the last ball wins the game. In his/her turn, it is mandatory to pick up at least one ball. A has to play first. What should he do to ensure a win?

SOLUTION

Deduction 1 The rules of the game define that there are two legal moves:
Picking up an equal number from each box or picking up any number from either box (at least 1).

Deduction 2 A has 17 possible moves to make and since the question asks for one particular move that will ensure a win, one of these 17 must be the winning move. It is at this stage that you should realise that the question calls for a comprehensive trial & error which should result in the elimination of 16 possibilities.
The starting position is 9, 4.
A's options at the start of the game can be basically divided into three options:

I	II	III
Pick up balls from the first box.	Pick up balls from the second box.	Pick up balls from both the boxes.

The position after A plays his move can be documented as follows (in terms of the number of balls B has in front of him.):

[There are 9 moves in option I.] (Balls from first box are picked by A.) If

(a) A picks up 1 ball, B will be left with 8, 4.
(b) A picks up 2 balls, B will be left with 7, 4
(c) A picks up 3 balls, B will be left with 6, 4
(d) A picks up 4 balls, B will be left with 5, 4
(e) A picks up 5 balls, B will be left with 4, 4
(f) A picks up 6 balls, B will be left with 3, 4
(g) A picks up 7 balls, B will be left with 2, 4
(h) A picks up 8 balls, B will be left with 1, 4
(i) A picks up 9 balls, B will be left with 0, 4

Similarly, if he picks up balls from the second box (option II) he will have an end result of: If

(a) A picks up 1 ball, B will be left with 9, 3
(b) A picks up 2 balls, B will be left with 9, 2
(c) A picks up 3 balls, B will be left with 9, 1
(d) A picks up 4 balls, B will be left with 9, 0

And for option III, If:

(a) A picks up 1 ball each from both boxes, B is left with 8, 3
(b) A picks up 2 balls each from both boxes, B is left with 7, 2
(c) A picks up 3 balls each from both boxes, B is left with 6, 1
(d) A picks up 4 balls each from both boxes, B is left with 5, 0

Out of these 17 options, the options of leaving (0, 4), (9, 0) and (5, 0) are infeasible, since it will result in an immediate win for B who can clean up the board in one move. Similarly, leaving (4, 4) will also cause an immediate win for B. *This leaves A with 13 options which he needs to consider. These are:*

I	II	III
8, 4	9, 3	8, 3
7, 4	9, 2	7, 2
6, 4	9, 1	6, 1
5, 4		
3, 4		
2, 4		
1, 4		

From these 13, the easiest option to check is (1, 4). *If B gets (1, 4) he has the following options:*

I	II	III
0, 4	1, 3	0, 3
	1, 2	
	1, 1	
	1, 0	

Obviously, he cannot play (0, 4), (1, 0), (1, 1) or (0, 3). This means he can leave A with (1, 2) or (1, 3). These need to be further investigated.

If B leaves (1, 2), A has the following options to leave for B:

I	II	III
0, 2	1, 1	0, 1
	1, 0	

Obviously, if he plays any of this, A will lose. Hence, if B leaves A with (1, 2), A will definitely lose.

Deduction 3 A cannot leave B with a situation in which B can make into (1, 2) or (2, 1).

Evaluating the 13 options left for A, we get that (1, 4), (2, 4), (3, 4), (5, 4), (9, 1), (9, 2), (6, 1) & (7, 2) are situations from which B can reach (2, 1) or (1, 2) in one move. This means B will win if A leaves him with any of these eight situations.

Thus, A will eliminate these eight options from his list of 13 and come down to five options which need further checking. These are: (6, 4), (7, 4), (8, 4), (9, 3) and (8, 3). At this stage we further need to eliminate four of these five options to come to the correct answer.

Now let us start our investigation with (6, 4). *If B is left with (6, 4) he has the following options:*

I	II	III
5, 4	6, 3	5, 3
4, 4	6, 2	4, 2
3, 4	6, 1	3, 1
2, 4	6, 0	2, 0
1, 4		
0, 4		

Of this, B cannot play (0,4), (1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 0), (6, 1), (6, 2), (2, 0), (3, 1) & (4, 2) [since these lead to a (2, 1) or a clean win in one move]. Thus, B can either leave (6, 3) or (5, 3).

If B leaves (5, 3) for A, A's options are:

I	II	III
4, 3	5, 2	4, 2
3, 3	5, 1	3, 1
2, 3	5, 0	2, 0
1, 3		
0, 3		

In each of these cases, B will either win in one move or get to (2, 1) in one move. Thus, A cannot do anything if B leaves (5, 3) in return for A leaving (6, 4).

Deduction 4 Just like being left with (2, 1) means a definite loss, so does being left with (5, 3).

Hence, A cannot allow a situation in which B can make it (5, 3) in one move.

Thus, A cannot leave B with (6, 4), (9, 3) or (8, 3) since, each of these will result in B giving back (5, 3) in one move.

Thus, A's option are down to (7, 4) or (8, 4). These need to be further investigated now:

Let us consider A leaving (7, 4) for B.

B's options are:

I	II	III
(6, 4)		
(5, 4)	(7, 3)	(6, 3)
(4, 4)	(7, 2)	(5, 2)
(4, 4)	(7, 1)	(4, 1)
(3, 4)	(7, 0)	(3, 0)
(2, 4)		
(1, 4)		
Or (0, 4)		

After closely evaluating each of these, all possible options will get eliminated because in one move they lead to either a straight win or a (2, 1) situation or a (5, 3) situation.

If A does any of this, then B cannot do anything to stop from losing the game.

The above question obviously is extremely long and complicated. However, what needs to be noticed is that even for this extremely long question, at no point of time is there any big logical jump, i.e., from one logic emerges the next one and so on.

Notes on Interpretation

Normally CAT questions only use 10% of this logic. However, let us now consider a question (Example 4) which appeared in CAT 2004 and was found to be extremely tough to crack. In fact, the question has been put down as unsolvable by most famous national level coaching centres on their website.

EXAMPLE 4 The year was 2006. All six teams in Pool A of World Cup Hockey play with each other exactly once. Each win earns a team three points, a draw earns one point and a loss earns zero point. The two teams with the highest points qualify for the semifinals. In case of a tie, the team with the highest goal difference (goals for–goals against) qualifies. In the opening match, Spain lost to Germany. After the second round (after each team played two matches), the Pool A table looked as shown on the next page.

In the third round, Spain played against Pakistan, Argentina played against Germany and New Zealand played against South Africa. All the third round matches were drawn. The following are some results from the fourth and fifth round matches.

(a) Spain won both the fourth and fifth round matches.

(b) Both Argentina and Germany won their fifth round matches by 3 goals to 0.

(c) Pakistan won both the fourth and fifth round matches by 1 goal to 0.

SOLUTION

For solving the above question, one requires tremendous clarity, logical consistency and above all a cool head.

Also, the solution of the above question is dependent on your ability to interpret the table and find out the appropriate linkages (something that might not be as easily done as said).

Let us look at the table and create our interpretations:

Deduction 1 Germany and Argentina have both won their matches while New Zealand and South Africa have lost both theirs.

Let us start by making tables for the possible result of the first two rounds for each of the six teams.

Germany

Round	Result
1. Spain	Won 1–0/2–1
2. Pak/New Zealand/South Africa	Won 2–1/1–0
3. Argentina	Drawn

It is given that Germany has played against Spain in the opening match (and obviously won that game).

Looking at the goals for & goals against columns and the fact that Germany has won both the first and second round matches, we deduce that its two wins must have been in one of the following combinations:

(Pool A)

Team	Games Played	Won	Drawn	Lost	Goals For	Goals Aganist	Points
Germany	2	2	0	0	3	1	6
Argentina	2	2	0	0	2	0	6
Spain	2	1	0	1	5	2	3
Pakistan	2	1	0	1	2	1	3
New Zealand	2	0	0	2	1	6	0
South Africa	2	0	0	2	1	4	0

Won 1 – 0	&	Won 2 – 1
	Or	
Won 2 – 1	&	Won 1 – 0

Notes on Interpretation

Germany cannot win either match by 2–0 or 3–0 or 3–1 margins since it will not be able to win the other match and maintain 3–1 goals for goals against situation.

Deduction If Germany had won the first game 2–1 against Spain, Spain would have won its second round match by 4–0, while if Germany won by 1–0, then Spain would have won its second round match 5–1 (since Spain has Goals For = 5 and Goals Against = 2).

Further, since only two teams — New Zealand and South Africa have conceded 4 or more goals, Spain must have played one of them. Looking into South Africa's Goals For/ Goals Against columns, if South Africa had conceded 4 goals in the second round, then it should have won the first round (1,0). But, South Africa has lost both rounds.

Hence, Spain played its second round against New Zealand. Further, if this is true, no other team can play against New Zealand in round two.

At this stage, the following possibilities emerge.

Team Germany

Round 1	vs.	Spain	Won	1–0	or	2–1
Round 2	vs.	Pak/S.A.	Won	2–1	or	1–0
Round 3	vs.	Argentina	Draw			

Team Spain

Round 1	vs.	Germany	Lost	0–1	or	1–2
Round 2	vs.	New Zealand	Won	5–1	or	4–0
Round 3	vs.	Pakistan	Draw			

Team New Zealand

Round 1	vs.	Arg/Pakistan	Lost	0–1	or	1–2
Round 2	vs.	Spain	Lost	1–5	or	0–4
Round 3	vs.	South Africa	Draw			

Deduction Team Pakistan won one round and lost one and Goals For/Goals Against 2/1. Hence, Team Pakistan won 2–0 and lost 0–1.

Now, since New Zealand played its first round against Pakistan or Argentina, it could not have lost 1–2. This is because in the case of Pakistan, if Pakistan had won 2–1 against New Zealand in round 1, its round 2 would have been a draw.

Further, Argentina has conceded no goals. Hence, it could not have won 2–1 against New Zealand.

This means that New Zealand must have lost 0–1 in its first match against Argentina (that cannot happen against Pakistan, because Pakistan cannot win 1–0 in the first round, since it will result in a 1–1 draw in round 2).

Consequently, New Zealand lost 1–5 in its second match to Spain and hence, Spain must have lost 0–1 to Germany.

The following scenario emerges from these deductions:

Team Germany

Round 1	vs.	Spain	Won	1–0
Round 2	vs.	S.A.	Won	2–1

Note: Here Germany's round 2 has to be vs. S.A., since Pakistan cannot lose 2–1

Team Spain

Round 1	vs.	Germany	Lost	0–1
Round 2	vs.	N.Z.	Won	5–1

Team New Zealand

Round 1	vs.	Argentina	Lost	0–1
Round 2	vs.	Spain	Lost	1–5

Team Pakistan

Round 1	vs.	S.A.	Won	2–0
Round 2	vs.	Argentina	Lost	0–1

The first three rounds are as under:

Round 1 matches:

Germany	beat	Spain	1–0
Argentina	beat	N.Z.	1–0
Pakistan	beat	S.A.	2–0

Round 2 matches:

Spain	beat	N.Z.	5–1
Argentina	beat	Pak	1–0
Germany	beat	S.A.	2–1

Putting all the deductions into one table, the following picture emerges:

	Germany	Argentina	Spain	Pak	New Zealand	S. Africa
Germany	—	D#3	W(1–0)#1			W(2–1)#
Argentina	D#3	—		W(1–0)#2	W(1–0)#1	
Spain	L(0–1)#1		—	D#3	W(9–1)#2	
Pakistan		L(0–1)#2	D#3	—		W(2–0)#1
New Zealand		L(0–1)#1	L(1–5)#2	—	D#3	
South Africa	L(1–2)#2			L(0–2)#1	D#3	—

According to the information available about the fourth and fifth round of matches:

Germany	—	Pakistan, lost (0–1) & N.Z. won (3–0)
Argentina	—	Spain, lost by 'x' goals & S.A. won (3,0)
Spain	—	Argentina won by 'x' goals & S.A. won by 'y' goals
Pakistan	—	Germany won (1–0) & N.Z. won 1–0
N.Z.	—	Germany lost (0–3) & Pakistan lost (0–1)
S.A.	—	Argentina lost (0–3) & Spain lost by 'y' goals

Germany, Argentina, Spain and Pakistan have all scored 10 points. Hence, the pool toppers would be determined on the basics of goal difference.

And the goal differences for the six teams are:

Germany	$+1+1+0-1+3=+4$
Argentina	$+1+1+0-x+3=5-x$ = Max. 4 or less
Spain	$-1+4+0+x+y=3+x+y$ = Min. 5 or more
Pakistan	$2-1+0+1+1=+3$
New Zealand	$-1-4+0-3-y=-8-y$

Based on these deductions, the following questions can be answered.

1. Which one of the following statements is true about the matches played in the first two rounds?
   (a) Pakistan beat South Africa by 2 goals to 1.
   (b) Argentina beat Pakistan by 1 goal to 0.
   (c) Germany beat Pakistan by 2 goals to 1.
   (d) Germany beat Spain by 2 goals to 1.
2. Which one of the following statements is true about the matches played in the first two rounds?
   (a) Germany beat New Zealand by 1 goal to 0.
   (b) Spain beat New Zealand by 4 goals to 0.
   (c) Spain beat South Africa by 2 goals to 0.
   (d) Germany beat South Africa by 2 goals to 1.
3. Which team finished at the top of the pool after five rounds of matches?
   (a) Argentina (b) Germany
   (c) Spain (d) Cannot be determined
   Spain must be top of the pool since it has the best goal difference even in it's worst case scenario.
4. If Pakistan qualified as one of the two teams from Pool A, which was the other team that qualified?
   (a) Argentina (b) Germany
   (c) Spain (d) Cannot be determined

Answers: 1. (b) 2. (d) 3. (c) 4. (d)

Notes on Interpretation

This question has an ambiguity. According to the deductions, Spain and Germany both should be above Pakistan in terms of goal difference and hence, Pakistan cannot qualify. However, if Pakistan qualifies, so do both Spain and Germany.

The above question was basically testing the ability of the student to analyse data. In the very same paper (CAT 2004), another question on data analysis went as follows:

EXAMPLE 5

Prof. Singh has been tracking the number of visitors to his homepage. His service provider has provided him with the following data on the country of origin of the visitors and the university they belong to:

Number of Visitors/Day

University	1	2	3
University 1	1	0	0
University 2	2	0	0
University 3	0	1	0
University 4	0	0	2
University 5	1	0	0
University 6	1	0	1
University 7	2	0	0
University 8	0	2	0

Number of Visitors/Day

Country	1	2	3
Canada	2	0	0
Netherlands	1	1	0
India	1	2	0
UK	2	0	2
USA	1	0	1

Deduction 1 Looking at Day 3, University 4 must belong to the UK and University 6 must belong to the USA.

Deduction 2 From Day 2 it is clear that University 8 has to be an Indian University while University 3 has to be from the Netherlands.

Deduction 3 From the analysis of Day 1 data, University 2 and University 7 should be distributed amongst the UK and Canada in either order, i.e. 2 belongs to the UK and 7 to Canada or 2 belongs to Canada and 7 to the UK. [Symbolically, (2 UK + 7 Canada) vs (2 Canada + 7 UK)]

Deduction 4 The visitor from USA on Day 1 must have come from University 6. Hence, University 1 and University 5 should be distributed between India and the Netherlands.

With this set of deductions, we get the following table. Using this table the answers to the following questions become quite elementary.

Number of Visitors/Day

University	1
University 1	I v N
University 2	UK v C
University 3	N
University 4	UK
University 5	N v I
University 6	USA
University 7	C v UK
University 8	I

1. To which country does University 5 belong?
   (a) India or the Netherlands but not the USA
   (b) India or the USA but not the Netherlands
   (c) The Netherlands or the USA but not India
   (d) India or the USA but not the UK
2. University 1 can belong to:
   (a) The UK (b) Canada
   (c) The Netherlands (d) The USA
3. Which among the listed countries can possibly host three of the eight listed universities?
   (a) None (b) Only the UK
   (c) Only India (d) Both India and the UK
4. Visitors from how many universities from the UK visited Prof. Singh's homepage in three days?
   (a) 1 (b) 2
   (c) 3 (d) 4

ANSWER KEY

1. (a) **2.** (c) **3.** (a) **4.** (b)

CHAPTER 2

Logical Reasoning Based on Arrangements

Arrangement questions are one of the most common question types in logical reasoning. As the name suggests, questions on arrangements typically involve arranging people or objects in straight lines, or around circles/squares, or other geometrical shapes.

The key skills involved in solving questions on arrangements include but are not limited to:

(i) the ability to visualise the geometrical shape of the arrangement situation.
(ii) the ability to order the clues in the correct order of usage (as explained in the theory of logical reasoning).
(iii) the ability to perceive what indirect clues are talking about–and how to use them.
(iv) the ability to convert clues written in language form into visual cues so that you do not need to read the text again and again. Also, converting the language clues to visual cues is critical for the purpose of being able to "see" all the clues at one go.

Illustrated below are the solutions to a few typical questions on arrangements. We would advise you to first have a look at the question and try to solve the same on your own before looking at the step-by-step process of solving the same– illustrated through the revolutionary "Reaction Tracker" mechanism, which is an integral part of this section of the book.

Note: The reaction tracker is a blow-by-blow account of exactly what reaction should go on in your mind as you solve an individual question in reasoning.

Look at the following questions and try to solve them:

EXAMPLE 1 Question at an easy level of difficulty

Directions: Study the information given below to answer these questions.

(i) Arnold's fitness schedule consists of cycling, rowing, gymnasium, jogging and boxing from Monday to Saturday, each workout is on one day, one day being a rest day.
(ii) Gymnasium workout is done neither on the first nor on the last day, but is done earlier than rowing.
(iii) Jogging is done on the immediate next day of the rowing day.
(iv) Cycling is done on the immediate previous day of the rest day.
(v) Jogging and boxing were done with a 2-day gap between them.
(vi) Boxing was done on the day following the rest day.

1. Which of the following is a rest day?
   (a) Wednesday (b) Tuesday
   (c) Friday (d) Thursday
2. Cycling and jogging days have a gap of how many days between them?
   (a) Nil (b) Two
   (c) Three (d) Four
3. On which day is boxing done?
   (a) Thursday (b) Friday
   (c) Monday (d) Wednesday
4. Which of the following is a wrong statement?
   (a) Gymnasium workout is done on the immediate previous day of rowing.
   (b) Jogging is done 3 days after the day on which boxing was done.
   (c) There is a gap of 3 days between the days on which cycling and rowing are done.
   (d) There is a 2 days' gap between the rest day and the day on which gymnasium workout is done.
5. Which of the following is the correct statement?
   (a) Jogging competition is done after rowing.
   (b) Cycling is done on Thursday.
   (c) No workout is done on Wednesday.
   (d) Rowing is done earlier than cycling.

REACTION TRACKER

The starting figure we start with when we read the first statement is:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday

Workouts are cycling, rowing, gymnasium, jogging and boxing.

From the second and third clues (gymnasium workout is done neither on the first nor on the last day, but was done earlier than rowing, and jogging was done on the day immediately following the rowing day), we know that rowing and jogging should be together and also that gymnasium has to be somewhere before this.

Visually this can be represented as:

G ⇨ [RJ]

From the fourth and sixth clues we have:

C-Rest day-B

Note: Putting it in a box signifies that there is no break between the items in the box.

Once we have these two visual representations we can go back to our original figure and think as follows:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday

Since gymnasium has to precede rowing and jogging, and gymnasium is not on the first day we can have three possibilities for placing gymnasium viz., Tuesday, Wednesday or Thursday.

Possibility 1:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	Gymnasium				

This case is rejected because once we place gymnasium we would need to place rowing and jogging in either **Wednesday–Thursday** or **Thursday–Friday** or **Friday–Saturday**. In each of these cases, we would also need to place a 3–day period having **Cycling–Rest day–Boxing.** It can be easily seen that in any of these three situations under Possibility 1, we do not have a completely free 3-day period anywhere in the week. Thus, we can reject Possibility 1.

Possibility 2:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		Gymnasium			

Rejected on the same logic as Possibility 1. There is no availability of a 3-day window to place **Cycling–Rest day–Boxing.**

Possibility 3:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			Gymnasium		

This is the only possibility that would work, as in this case, the respective work-out, ordering would be:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Cycling	Rest Day	Boxing	Gymnasium	Rowing	Jogging

Thus, the answers are:

1. (b); Tuesday is the rest day.
2. (d); 4 days
3. (d); Boxing is done on Wednesday.
4. (d); The statement in Option (d) is incorrect.
5. (a); Only the statement in Option (a) is correct.

EXAMPLE 2

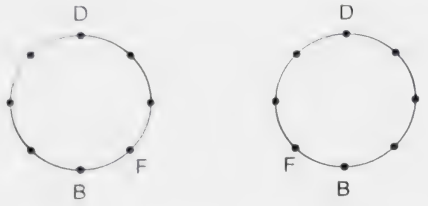
Directions: Study the following information to answer these questions.

(i) Alex, Betsy, Chloe, Dennis, Edward, Fiona, Giles and Herbert are gamblers sitting around a round table facing the center.
(ii) Dennis is the neighbour of Alex but not of Herbert.
(iii) Betsy is the neighbour of Fiona and 4th to the left of Dennis.
(iv) Edward is the neighbour of Herbert and 3rd to the right of Fiona.
(v) Chloe is neither the neighbour of Alex nor of Giles.

1. Which of the following is wrong?
   (a) Betsy is to the immediate left of Herbert.
   (b) Herbert is to the immediate left of Edward.
   (c) Dennis is 4th to the right of Fiona.
   (d) All are correct.
2. Which of the following is correct?
   (a) Dennis is to the immediate left of Giles.
   (b) Alex is between Chloe and Dennis.
   (c) Fiona is 3rd to the right of Dennis.
   (d) Edward is between Herbert and Betsy.
3. Which of the following groups has the 2nd person sitting between the 1st and the 3rd?
   (a) Alex–Fiona–Chloe
   (b) Giles–Alex–Dennis
   (c) Betsy–Edward–Herbert
   (d) Herbert–Fiona–Betsy
4. Which of the following pairs has the 1st person sitting to the immediate right of the second?
   (a) Betsy–Herbert (b) Fiona–Betsy
   (c) Edwards–Giles (d) Alex–Dennis
5. Which of the following pairs are fourth to one another?
   (a) Chloe–Edwards (b) Fiona–Herbert
   (c) Dennis–Chloe (d) Dennis–Betsy
6. If Chloe and Giles interchange their positions, which of the following will indicate Alex's position?

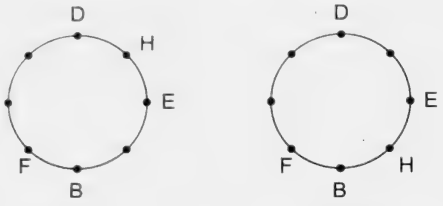
(a) To the immediate right of Giles
(b) 4th to the right of Chloe
(c) 2nd to the left of Giles
(d) To the immediate left of Chloe

Solution for Example 2: From clues (i) and (iii), we get following two possibilities:



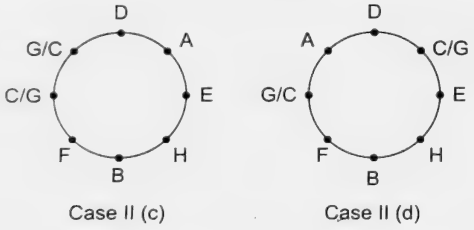
Case I Case II

Now, read clue (iv): Edward is third to the right of Fiona. This information rejects Case I. Using clue (iv), we get the following two possibilities, i.e., Case II (a) and Case II (b).



Case II (a) Case II (b)

Now, from clue (ii), we come to know that Dennis is not the neighbour of Herbert. Hence, reject Case II(a). From clue (ii), we get the following two possibilities:



Case II (c) Case II (d)

Now, from clue (v), Chloe cannot be the neighbour of Giles.

REACTION TRACKER

Let us arrange the whole information.
From clue (i), we get

Khurusheva | 7 Soldiers | Lambert

Now, using clue (iii), we get

Jackson | 7 Soldiers | Khurusheva | 7 Soldiers | Lambert | 10 Soldiers

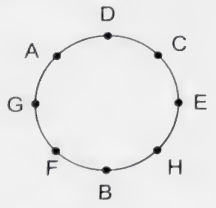
Now, using clue (iv) and clue (v), we get

Jackson | 1 Soldier | Mickey | 5 Soldiers | Khurusheva | 7 Soldiers | Lambert | 10 Soldiers

Now, from clue (ii), we get the complete information as following:

13 Soldiers | Jackson | 1 Soldier | Mickey | 5 Soldier | Khurusheva | 7 Soldiers | Lambert | 10 Soldiers

Therefore, reject Case II (c).
Again, since Chloe cannot be the neighbour of Alex, the final seating arrangement will be as follows:



The answers then become pretty straight forward.

1. (c)	2. (c)	3. (b)	4. (d)	5. (d)	6. (d)

EXAMPLE 3

Directions: These questions are based on the information that follows.
In a row of soldiers facing North, (i) Lambert is 8th to the right of Khurusheva; (ii) Mickey is 16th from the left end; (iii) Lambert is 16th to the right of Jackson, who is 27th from the right end of the row; (iv) Khurusheva is nearer than Mickey to the right end of the row; (v) there are five boys between Mickey and Khurusheva.

1. How many soldiers are there between Jackson and Mickey?
   (a) One (b) Two
   (c) Three (d) Data inadequate
2. How far away is Khurusheva from the right end of the row?
   (a) 30th (b) 10th
   (c) 19th (d) 18th
3. How many soldiers are there in the row?
   (a) 50 (b) 40
   (c) 36 (d) Data inadequate
4. How far away is Jackson from the right end of the row?
   (a) Data inadequate (b) 24th
   (c) 25th (d) 27th

CHAPTER 3

Logical Reasoning Based on Rankings

Reasoning questions on rankings involve an ordering of people objects based on their heights/weights/performance in an exam, etc. As the name suggests, in questions on rankings you are supposed to place people/objects in a decreasing or increasing order based on an attribute being measured.

Key skills required in solving logical reasoning questions based on rankings:

(i) The ability to visualise the structure in which the rankings have to be created

(ii) The ability to order the clues in the correct order of usage (as explained in the theory of logical reasoning)

(iii) The ability to perceive what indirect clues are talking about, and find the appropriate point in the solving process about how to use them

(iv) The ability to convert clues written in language form into visual cues so that you do not need to read the text again and again, and also are able to "see" all the clues at one go.

The following illustrated examples (with the reaction tracker used to explain the solutions) would help you get acquainted with questions based on rankings. For each of the following questions first try to solve them on your own before looking at the reaction tracker process of solving the same.

Example 1

Directions for Questions 1–4 (Constraint-based Arrangement):

i. Six students A, B, C, D, E and F participated in a self-evaluation test of Quants and Data Interpretation (D.I.).

ii. The total marks of A in Quants was just above C and in D.I. just above F.

iii. B was just above C in D.I., but he scored less than D in Quants.

iv. F got more marks than D and E in D.I. but did not perform as well in Quants as in D.I as compared to D and E.

v. No one is in between C and D in Quant and C and A in D.I.

1. Who got the highest marks in D.I.?
   (a) A
   (b) B
   (c) C
   (d) Data inadequate
2. Which of the following students has scored the least in D.I.?
   (a) Only D (b) Only E
   (c) Only D or E (d) None of these
3. Who was just below D in Quants?
   (a) B (b) E
   (c) C (d) Data inadequate
4. Which of the given statements is not necessary to answer the questions?
   (a) (ii) (b) (iii)
   (c) (iv) (d) All are necessary

REACTION TRACKER

From the second statement we have:

QUANTS	D.I.
A	A
C	F

From statement (iii) we have: B just above C in D.I. and B somewhere below D in Quants. At this point our figure remains the same as we cannot put this information into the figure.

QUANTS	D.I.
A	A
C	F

From statement (iv) we have:

QUANTS	D.I.
A	A
C	F
D and E above F	D and E below F

From Statement (v) we have:

QUANTS	D.I.
A	B
C	C
D	A
	F
	D and E below F

This leads us to the following table:

In Quants	In Data Interpretation
A	B
C	C
D	A
	F
Also, B is less than D.	D and E are below F. Hence B must be first.
F is below D and E. Note: E could be placed anywhere as we do not have any information about E.	

The answers are:

1. B (b)
2. D or E (c)
3. Data inadequate (d)
4. All are necessary (d)

EXAMPLE 2 Solve the following questions based on the information provided:

i. A, B, C, D, E, F, G and H are eight friends. Three of them play cricket and table tennis each, and two of them play football. Each one of them has different height.
ii. The tallest does not play football and the shortest does not play cricket.
iii. F is taller than A and D but shorter than H and B.
iv. E, who does not play cricket, is taller than B and is second to the tallest.
v. G is shorter than D but taller than A.
vi. H, who is fourth from the top, plays table tennis with D.
vii. G does not play either cricket or football. B does not play football.

1. Who is the tallest?
   (a) B (b) H
   (c) C (d) Data inadequate
2. Who is the shortest?
   (a) G (b) D
   (c) A (d) Data inadequate
3. Which of the following pairs of friends plays football?
   (a) EA (b) EH
   (c) HF (d) Data inadequate
4. What is F's position from the top when they are arranged in descending order of their height?
   (a) Fifth (b) Fourth
   (c) Sixth (d) None of these

REACTION TRACKER

The first direct clue in the question is clue (vi). Clue (vii) is the next clue to use as using the last two clues we get that H,G and D play table tennis (thus fixing our list of TT players). Hence, when we read that B does not play football we can deduce that B plays cricket—as he cannot be playing table tennis since we already know the three people playing TT. The other direct clue that we can use for placing an individual in our starting figure based on height is clue (iv). According to this, E does not play cricket (hence must be football) and he is second to the tallest. At this point we also need to keep aside the information that E is taller than B outside the purview of the figure for later use. At this point our figure would be as given below with the additional knowledge that H,G and D play TT and B plays cricket.

–	–
E	Football
–	–
H	Table tennis
–	–
–	–
–	–
–	–

From this point we need to first focus on placing the other six people in the decreasing order of their heights inside the figure above.

clue (iii) combined with clue (v) helps us understand that:

Between A, G and D, the order of heights from lower to higher would be A → G → D.

And since F is taller than A and D, F must be taller than A → G → D. Also because F is shorter than H and B, F cannot take the top most or the third position from top in the figure. Based on these deductions the figure changes to:

–	–
E	Football
–	–
H	Table tennis
F	–
D	Table tennis
G	Table tennis
A	–

Placing B in the above figure is easy because we had kept aside the information that "E is taller than B". Thus, B can

only take the third place position and the highest place goes to C. Thus the figure becomes:

C	–
E	Football
B	Cricket
H	Table tennis
F	–
D	Table tennis
G	Table tennis
A	–

At this point we can use the clues about who does not play what and identify the remaining two people for cricket and the remaining person for football as:
The tallest does not play football → deduction → the tallest plays cricket.
The shortest does not play cricket → deduction → the shortest plays football. The table can be completed at this point as below:

C	Cricket
E	Football
B	Cricket
H	Table tennis
F	Cricket
D	Table tennis
G	Table tennis
A	Football

The answers can be read off easily at this point from the table above:

1. C is the tallest. Option (c).
2. A is the shortest. Option (c).
3. E and A are the two football players. Option (a).
4. F is placed fifth from the top. Option (a).

CHAPTER 4

Team Formation

Team formation questions are another question type that are commonly seen in most exams. The questions consist typically of a group of people/objects that have to be put together in teams— albeit with a set of constraints/ conditions about certain people who need to be put together/ cannot be put together in a team.

The key skills involved in solving team formation questions would involve the following:

(i) The ability to visualise the structure of the teams to be formed including the number of people/objects in each team

(ii) The ability to order the clues in the correct order of usage (as explained in the theory of logical reasoning)

(iii) The ability to create symbolic representations of the various clues provided so that you can bring together each of the relevant clues while creating the teams

(iv) The ability to wait for and reach the appropriate time in the problem-solving situation from where the indirect clues provided in the question can be used.

Illustrated below are the solutions to a few typical questions on team formations. We would urge you to first have a look at the questions and try to solve the same on your own before looking at the solutions.

EXAMPLE 1

Directions for Questions 1–2: Read the information given below and answer the questions.

A director is casting a movie about twins.Selection must be made from among nine people—Amartya, Bhupesh, Caruna, Divya, Elangovan, Farly, Girish, Harish and Isha. Amartya is Bhupesh's twin, Caruna is Divya's twin, and Elangovan is Farly's twin. Girish, Harish and Isha may each be selected only as "extras."

- At least two pairs of twins must be selected.
- At least as many women as men must be selected.
- Amartya and Bhupesh must both be selected, if either is selected.
- Caruna and Divya must be selected, if either is selected.
- Elangovan and Farly need not both be selected.
- At least one "extra" must be selected.
- Amartya, Bhupesh, Elangovan, Girish and Harish are men.

1. Which of the following is an acceptable cast for the movie?
   (a) Amartya, Bhupesh, Elangovan, Farly, Isha
   (b) Caruna, Divya, Farly, Girish, Harish, Isha
   (c) Bhupesh, Caruna, Divya, Elangovan, Harish, Isha
   (d) Caruna, Divya, Elangovan, Farly, Girish, Harish, Isha
2. Which of the following people must be included in the cast?
   (a) Divya (b) Isha
   (c) Farly (d) Harish

SOLUTION

To solve this we will create a diagram where men will be denoted by upper case letter and women with lower case letters and we will list the extras separately to avoid confusion. Here given that Caruna and Divya must both be selected because if they were not selected then Amartya and Bhupesh or Elangovan and Farly (three of whom are men) would all be selected; as a result only two women Farly and Isha at most will be selected. However according to the constraints at least as many women as men must be selected; thus Caruna and Divya must both be selected.

[A B] [ⓒ ⓓ] [E / F]

G H i [≥ 1]

women ≥ men

1) Eliminate each options using the clues given in the information. Only option (d) suffices. Hence option (d) is the correct answer.
2) Caruna and Divya must both be selected. Hence option (a) is the correct answer.

EXAMPLE 2 Vijayshree wants to take four courses this trimester. There are only seven courses in which she is interested: three Marketing courses – Distribution, Advertising and PR; and four Finance courses—International Finance, Accounting, Corporate finance and Financial Services. To meet college requirements she must take two marketing courses. There are some scheduling problems: International Finance overlaps both Advertising and Corporate Finance, but she can choose Advertising and Corporate Finance as two different courses. Distribution is given at the same time as Accounting.

1. If Vijayshree decides she will take International Finance, what will her other three courses be?
   (a) Distribution, PR and Advertising
   (b) Distribution, PR and Financial Services
   (c) Distribution, PR and Accounting
   (d) PR, Advertising and Financial Services
2. If Vijayshree takes four courses this trimester, then which of the following statement is not possible?
   I. She takes Accounting and does not take Advertising.
   II. She takes Corporate Finance and does not take Advertising.
   III. She takes International Finance and does not take PR.
   (a) I only (b) II only
   (c) III only (d) I and III only
3. Which of the following must always be true?
   I. Vijayshree must take PR if she takes Corporate Finance.
   II. Vijayshree must take Advertising if she takes Accounting.
   III. Vijayshree must take Accounting if she takes Advertising.
   (a) I, II, and III (b) II, and III only
   (c) I and II only (d) II only

SOLUTION

(1) It can be seen that if she takes International Finance, she would not be able to take the Advertising and Corporate Finance course. This means that she has to choose Distribution and PR as her two marketing courses, and because Distribution overlaps with Accounting, her second finance course must be Financial Services. Option (b) is correct.

(2) Statement I is definitely not possible because if she takes Accounting she cannot take Distribution and she must have to take Advertising.
Statement II is possible, while statement III can be seen to not be possible as if she takes International Finance she has to take PR as Advertising would not be available to her.
Option (d) is correct.

(3) Statement I is not necessarily true as if she takes Corporate Finance she can take up Distribution and Advertising and skip PR. i.e., PR is not necessary if she has taken Corporate Finance.
Statement II is mandatorily true because if she takes Accounting, she cannot take Distribution and hence she would be forced to take up Advertising as one of her two compulsory marketing courses. Statement III is not necessarily true. Thus option (d) is correct.

EXAMPLE 3

Directions for Questions 1–3: Answer the questions based on the following information.

At Semco, Ricardo Semlar has perfected the art of employee motivation. Due to this, his staff works 7 days a week. However, he has given them the leeway that in every week they are allowed to work for the company on any 4 days and for the remaining 3 days of the week every employee is allowed to work on his own entrepreneurial project. On a particular day, Mr. Semlar was looking closely at the functioning of three of his most trusted executives and found out a few facts about their weekly schedule.

Each of them work for the company only four days a week and work on their own projects for three days every week.
All the three executives work together (for the company) only once in a week.
None of them works for the company for three consecutive days.
Amit works on his project on Tuesdays, Thursdays and Sundays.
Bimlesh works on his own project on Saturday.
Chetan does not work for the company on Fridays and Sundays.
No two executives have an own project work day on the same day more than once a week.
At least 1 person works for the company everyday of the week.

1. On which day of the week do all the three executives work together?
   (a) Monday (b) Wednesday
   (c) Friday (d) Cannot be determined
2. Which of the following days does Chetan work on his own project?
   (a) Monday (b) Tuesday
   (c) Wednesday (d) Thursday
3. Which of the following days is surely a company working day for Bimlesh?

(a) Wednesday (b) Friday
(c) Sunday (d) Cannot be determined

SOLUTION

The following table emerges from the given conditions: (P denotes working on own project)

	Mon-day	Tues-day	Wednes-day	Thurs-day	Fri-day	Sat-urday	Sun-day
Amit		P		P			P
Bimlesh						P	
Chetan			P		P		P

Note: Chetan's third P is on Wednesday because it cannot be on Tuesday or Thursday (as in that case Chetan and Amit would be working on their projects together on two days of the week.) Also, Chetan's third P cannot be on Monday in which case Chetan would be working continuously for the company for three days.

Thus, the answers are:

1. Option (a) (Monday)
2. Option (c) (Wednesday)
3. Sunday is necessarily a company working day for Bimlesh. Option (c) is correct.

EXAMPLE 4 Study the following information carefully and answer the questions given below:

seven friends A, B, C, D, L, M and Z are going to a new year's party on mobikes in Goa. Since it is late at night they do not anticipate any police presence and hence have taken only three bikes – an Enfield, a Honda and a TVS, with at least two of them sitting on each bike (hence there is triple riding on at least one bike). There is exactly one male on each bike. Amongst the group there are two executives, two designers and three psychologists among them.

(i) C is a lady designer and she does not travel with the pair of sisters, A and M.
(ii) B, a male executive, travels with only Z, a psychologist on an Enfield bike.
(iii) D is a male designer.
(iv) Two persons belonging to the same profession do not travel on the same bike.
(v) A is not an executive and travels on the Honda.

1. What is M's profession?
   (a) Executive (b) Psychologist
   (c) Designer (d) Data inadequate
2. On which bike does C travel?
   (a) Enfield (b) Honda
   (c) TVS (d) Either Honda or TVS
3. Which of the following represents the three psychologists?
   (a) ZLM (b) ZLA
   (c) ZLM or ZLA (d) None of these

SOLUTION

We start from clue (ii) and get the following table:

Enfield	Honda	TVS
B Z Executive – Psychologist		

(Note we can conclude that Z is a female because there is exactly one male on each bike). We denote females in our figures with an underline.

We further know that A and M are sisters and C is a lady designer. Since two persons with the same profession cannot travel together, D and C must be traveling on different bikes. Thus, D should be clubbed with A and M. Also, Since A is on a Honda, the Honda should have 3 people A, M and D.

Enfield	Honda	TVS
B Z Executive – Psychologist	A M D ?, ?, Designer	C L Designer, ?

From this point in the solution, we need to concentrate only on the professions of A, M and L. We also know that we are yet to identify one executive and two psycholoigsts. Further, according to the constraint of clue (iv) both A and M cannot be psychologists. Thus, L must be a psychologist. Also, since, we know that A is not an executive we can conclude that A must be a psychologist and the final table emerges as follows:

Enfield	Honda	TVS
B Z Executive – Psychologist	A M D Psychologist, Executive, Designer	C L Designer, Psychologist,

The answers are:

1. Executive. Option (a) is correct.
2. On the TVS. Option (c) is correct.
3. Z,L and A represent the three psychologists. Option (b) is correct.

EXAMPLE 5

Direction for Questions 1–3: Study the following information carefully to answer the questions given below:

Examinations of eight papers were conducted in a week, from Monday to Saturday. The papers were: Advertising, Biology, Chemistry, Distribution, Quantitative Techniques, Finance, Marketing and Heuristics. Not more than two papers were organised in a day. Only one of the days, during Monday to Saturday, was the rest day. The paper

on Distribution was held just before the Finance paper, but immediately after the Heuristics paper. It is also known that there was no rest day between any two of these three papers. The tests were split equally between the days before the rest day and the days after the rest day, i.e., tests of four papers were held before the rest day whereas four papers were held after the rest day. Thursday was not the rest day. Quantitative Techniques and Finance were held on the same day. The paper on Heuristics was not held either on Thursday or on Friday. The papers on Marketing and Biology were held just before Advertising and Chemistry, respectively. The paper on Advertising was held just before the paper on Biology.

1. Examinations of which papers were held on Monday?
   (a) Heuristics and Distribution
   (b) Distribution and Finance
   (c) Marketing and Advertising
   (d) Cannot be determined
2. Which of the following days was the rest day?
   (a) Tuesday (b) Wednesday
   (c) Thursday (d) Cannot be determined
3. Examination(s) of which of the following papers was /were held on Friday?
   I. Marketing II. Advertising
   III. Biology IV. Chemistry
   (a) Only II
   (b) Only III
   (c) Either II or III or both II and III
   (d) Both II and III

SOLUTION

The thinking in this question would go as follows:

From the statements "Examinations of eight papers were conducted in a week, from Monday to Saturday. Not more than two papers were organised in a day and only one of the days, during Monday to Saturday, was the rest day." we realise:

Eight papers are conducted in five days with not more than 2 papers on any single day. This means that there must be exactly three days when two papers each are conducted.

Further when we read "Tests of four papers were held before the rest day whereas four papers were held after the rest day", we realise that there must be at least two days before the rest day and two days after the rest day. This obviously means that the rest day must be either on Wednesday or on Thursday.

Also as we go further down the problem we realise: "Thursday was not the rest day." We realise that the rest day must be on Wednesday. Thus, both Monday and Tuesday must have had two exams each.

From the statements: "The papers on Marketing and Biology were held just before Advertising and Chemistry, respectively. The paper on Advertising was held just before the paper on Biology." We get that **Marketing–Advertising–Biology–Chemistry** must be the order for these 4 subjects and also that there must not be any other paper between these four papers.

At this point we know the following structure of the respective days and the papers on each day:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
2 Papers	2 Papers	REST DAY			

In the above figure we also know that between Thursday to Saturday exactly one day has two papers. Thus, there could be two papers on any one of the days viz: Thursday, Friday or Saturday.

Further using the statements "The paper on Distribution was held just before the Finance paper, but immediately after the Heuristics paper. It is also known that there was no rest day between any two of these three papers and Quantitative Techniques and Finance were held on the same day. The paper on Heuristics was not held either on Thursday or on Friday." We get:

Heuristics–Distribution–Finance–Quantitative Techniques as one order of papers which can either be placed before the rest day or after. However, if we were to try to place these four papers after the rest day, we would need to put Heuristics on Thursday, which contradicts the conditions of the problem. Thus, these four papers can only be assigned to Monday and Tuesday (two papers each). This also means that: **Marketing–Advertising–Biology–Chemistry** must be after the rest day.

Based on these conclusions there are three possible ways in which the exams can be structured (depending on which day we use for putting two papers after Wednesday). These are

Possibility 1:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Heuristics	Finance	REST DAY	Marketing	Advertising	Chemistry
Distribution	QT			Biology	

Possibility 2:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Heuristics	Finance	REST DAY	Marketing	Advertising	Biology
Distribution	QT				Chemistry

Possibility 3:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Heuristics	Finance	REST DAY	Marketing	Biology	Chemistry
Distribution	QT		Advertising		

The answers can be read off from the tables (based on what is correct for each of these possibilities):

1. Heuristics and distribution are always on Monday. Option (a) is correct.
2. Wednesday is the rest day in all possibilities. Option (b) is correct.
3. It could be either Biology (possibility 3), or Advertising (possibility 2) or both (possibility 1). Option (c) is correct.

CHAPTER 5

Quantitative Reasoning

Quantitative reasoning, as the name itself suggests, is reasoning involving numbers and numerical logic. Quantitative reasoning questions are common in all kinds of aptitude exams and require the student to work out numerical relationships, as defined by the parameters in the problem.

The key skills involved in solving quantitative reasoning questions are the following:

(i) The ability to understand the specific quantitative logic being utilised inside the questions
(ii) The ability to order the clues in the correct order of usage (as explained in the theory of logical reasoning)
(iii) The ability to understand basic mathematical concepts like percentages, averages, ratios. multiplications, etc.
(iv) The ability to create symbolic representations of the various clues provided so that you can bring together each of the relevant clues while creating the teams
(v) The ability to wait for and reach the appropriate time in the problem-solving situation from where the indirect clues provided in the question can be used.

Illustrated below are the solutions to a few typical questions on quantitative reasoning. We would urge you to first have a look at the questions and try to solve the same on your own before looking at the solutions.

EXAMPLE 1 The classic quantitative reasoning question:

A man would like to take a new health insurance. An officer taking care of these matters says to the man, "Please tell me how many children you have." The man answers, "I have three of them." The officer, "What are the ages of your children?" The man answers, "The product of the ages is equal to 36." The officer replies, "This is not enough information Sir!"; the man replied, "Sorry that I was a little bit unclear, but the sum of the ages is equal to the number of shops in front of your office." The officer: "This still is not enough information Sir!"; The man replies, "My oldest child loves chocolate." The officer: "Thanks for your cooperation, I now know the ages." Are you as smart as the officer? Then give the sum of the ages of the children.

(a) 13 (b) 22
(c) 36* (d) 38

➤ **SOLUTION**

The product of the ages is. Using this one can make the following combination of ages:

1,36, 1 sum = 38
1,18, 2 sum = 22
1,12, 3 sum = 16
1, 9, 4 sum = 14
1, 6, 6 sum = 13
2, 9, 2 sum = 13
2, 6, 3 sum = 11
3, 3, 4 sum = 10

After the man had said that the product of the ages is equal to 36, the officer did not have enough information. Then he was told that the sum is equal to number of shops in front of the office. He replied by saying that this still is not enough information. So the sum of the ages should be 13, because otherwise he would have known the ages immediately. The last statement is that that the oldest child loves chocolate. So there is an oldest child. Hence, the officer concludes that the ages of the children are 2, 2 and 9 years.

Hence, option (c) is the correct answer.

EXAMPLE 2 Early one Monday morning, four snails—Aman, Bubbly, Charu and Devi, set off together down the garden path. Aman and Bubbly kept the same steady pace, slithering only 8 meters by the time Charu and Devi had already reached the azalea.

Charu was winded and had to stop for an hour to rest. Although Devi was tired, too, she pressed on, but reduced her pace to be the same as Aman's and Bubbly's.

Charu started off again just as Aman and Bubbly got even

with her. She raced off at her original pace. Aman promptly sped up to the same speed as Charu and kept even with her. Bubbly just kept going at her original pace.

When Aman reached the end of the path, she was 1 meter ahead of Bubbly, but she was a half hour late than Devi was.

How many meters long was the path?

(a) 10 (b) 15

(c) 8 (d) None of these

SOLUTION

A few things are evident from the current situation:

Deduction i): Since Charu rests for 1 hour before Aman and Bubbly reach the azalea, it is clear that Devi would be exactly 1 hour ahead of Bubbly when she reaches the end point.

Deduction ii): Since Devi has already reached half an hour ago when Aman and Charu reach, Bubbly must be only half an hour from her destination. Since, Bubbly is 1 meter away from the destination, she would cover this 1 meter in 30 minutes. Thus, Bubbly's speed must be 2 m/h. This would be the slower of the two speeds.

This would also be the speed of Bubbly and Aman for the first 8 meters.

Deduction iii): Since, Bubbly and Aman take 4 hours to reach 8 meters, in 1 hour more (when Charu rests) they will cover 2 meters more. Thus, the azalea is at a distance of 10 meters from the starting point.

Deduction iv): The 10 meter distance to the azalea is covered in 4 hours by Charu and Devi. Thus, the faster speed would be 2.5 m/h.

Deduction v) After the azalea, when Aman and Charu start moving at 2.5 m/h, Bubbly would be moving at 2 m/h. When Aman and Charu reach the end, Bubbly is 1 meter behind. This gap can only be created in a 2 hour journey.

Thus, the distance from the azalea to the end point is 5 meters. Total distance is 10+5=15 meters.

EXAMPLE 3 Dhiman was admiring the output of her new program to generate random numbers. She had printed out the first ten numbers of the results. She soon noticed something interesting. Each of the 10 numbers had exactly one digit, in the proper placement, of the 5-digit code she used to open her car door without a key.

In the first number 14073, for example, Dhiman's car code could not be 34170 (two digits correctly placed) or 92365 (none).

Find Dhiman's car entry code from these first 10 randomly generated numbers: 14073, 79588, 05892, 84771, 63136, 42936, 37145, 50811, 98174 and 29402?

(a) 05892 (b) 63136

(c) 42936 (d) None of these

SOLUTION

In the given grid of 10 five-digit numbers, since every number has exactly 1 digit matched correctly with the correct code, there must be exactly 10 instances of correct code matches amongst the 50 possible instances (10 × 5).

If we were to look at the first digit it is clear that the first digit has 10 different values in the 10 numbers. Thus, only 1 number can be correctly matched for the first digit's value.

The other four places in the 5-digit number must match nine more times for the above grid to be correct. This can be principally done in 2 ways:

First way: 3 + 3 + 2 + 1 OR Second way: 2 + 2 + 2 + 3

If we were to observe the numbers the following deductions would come up:

First place	Second place	Third place	Fourth place	Fifth place
Each digit (from 0 to 9) happens only once	4 and 9 occur twice	1 occurs thrice and 8 occurs twice	7 occurs thrice and 3 occurs twice	1,2 and 6 occur twice each
	5,3,2,7,0 and 8 occur once each	0,5,7,9,8 occur once	8,9,4,1 and 0 occur once each	5,4,3 and 8 occur once each

Looking at the number 98174, it is clear that both 1 and 7 cannot occur in the third and fourth place digits as then the number 98174 would have two numbers matched with the code.

Thus, the structure 1+2+3+3+1 or indeed 1+1+3+3+2 is ruled out (i.e., the use of two triplicate matches is ruled out). Thus, the 10 matches between the code and the 10 random numbers must be in one of the following structures:

	First Place	Second Place	Third Place	Fourth Place	Fifth Place
Number of digits matched with the code	1	2	3	2	2
Number of digits matched with the code	1	2	2	3	2

Thus, the number would be either xx13x or xx87x. The first possibility is rejected because 63136 contains both 1 and 3 in the third and fourth places of the number. Thus, the number must be of the form xx87x. From this point thinking about the second place digit gives us that the second place has to be occupied by 9 (as only 4 and 9 give us two matches with the code and amongst this 4 cannot be taken for the second place because if the number is x487x then the number 84771 contradicts the basic condition of the problem.)

Thus, the number must be x987x.
Thinking of the last digit – this digit must be either 1, 2 or 6. It cannot be 1 because of the presence of 84771 in the ten numbers. It cannot be 2 because the number 29402 would have two digits in the correct place. Hence, it must be 6. Thus, the number becomes x9876.
Thinking of the first digit, it has to be 3, because it is only the number 37145 which lacks a single digit matching with the 2nd to 5th digit of the code x9876. Thus, the code is 39876.

EXAMPLE 4 Abe, Buddy, Carmen, Dennis and Earl all live on Pine Street, which has house numbers from 10 to 111, both inclusive. Two of them live in the same house. The others all live in different houses. They all have made remarks about where they live, but not all the remarks are true.
Abe said, "My house number is a factor of Buddy's house number. Earl's house number is 10, greater than Dennis'."
Buddy said, "My house number is greater than 70. Abe's house number is greater than 30".
Carmen said, "My house number is both a cube and a square. Dennis's house number is greater than 50".
Dennis said, "My house number is a square. Buddy's house number is a cube".
Earl said, "My house number is twice Buddy's".
But who's telling the truth? It turns out that all statements made by people living in houses with numbers greater than 50 were false. All the other statements were true.

Can you tell the house number of Earl?

(a) 49 (b) 16
(c) 59 (d) None of these

SOLUTION

In order to solve this question you need to look at the options for Earl's house number. Option (a) cannot be true since if Earl's house number is 49, he must be speaking the truth – but his statement cannot be true. If we take Earl's house number as 16 (Option (b)), then Buddy must be 8, which is against the problem's basic condition that all house numbers are between 10 and 111.

If we go with Option (c)—59, then if Abe is true then Dennis must be 49. Hence, Dennis's statements must be true. So Buddy's house number would be a cube. There are only two perfect cubes between 10 and 111— viz 27 and 64. If Buddy's house number is 27, then he must be speaking the truth—which is not true if we look at the statement of the problem. Thus, Buddy must be 64 and Abe's house number must be less than 30 – only factor of 64 less than 30 and greater than 10 is 16.

At this time we have the grid as:

Abe	16
Buddy	64
Earl	59
Dennis	49

We just need to place Carmen in this grid. Looking at Carmen's statement and looking at the numbers available below 50, the two numbers (16 and 49) do not satisfy the condition that the room number is both a square and a cube. Hence, Carmen must be lying, and the possible values of Carmen is 59 and 64. If we were to place Carmen at 64, then Carmen's first statement becomes true—an internal contradiction. Thus, Carmen must be at 59. The final arrangements of house numbers are:

Abe	16
Buddy	64
Carmen and Earl	59
Dennis	49

EXAMPLE 5 During a game of five card draw poker, played with a standard deck, you are dealt a hand with the following characteristics:

- It contains no aces or face cards.
- No two cards have the same value.
- All four suits are present.
- The total value of the odd cards equals the total value of the even cards.
- There are no three straights cards.
- The total value of the black cards is 10.
- The total value of the hearts is 14.
- The card with the lowest value is a spade.

Exactly what are the five cards in your hand?

(a) 2 of spades, 5 and 9 of diamonds, 4 of spades and 8 of clubs
(b) 2 of diamonds, 5 and 9 of spades, 4 of hearts and 8 of clubs
(c) 2 of clubs, 5 and 9 of hearts, 4 of diamonds and 8 of spades
(d) 2 of spades, 5 and 9 of hearts, 4 of diamonds and 8 of clubs

SOLUTION

Deduction i): Since there are no aces or face cards, we only have 2, 3, 4, 5, 6, 7, 8, 9 or 10 as the value of the cards.

Deduction ii) Since total of odd = total of evens and the minimum even total is 12 (2 + 4 + 6), there must be two odds and three evens. Only then can the two odds add up to an even total and be equal to the total of 3 even cards.

Deduction iii) There are two cards of 1 suit and one card each of the other 3 suits. The two cards belonging to 1 suit must be hearts because there is no other way for the hearts to total up to 14.

Deduction iv) There are three cases:

a) If the sum of the evens is 12, the odds would be 5 and 7 and the evens would be 2, 4, 6. This cannot happen because in such a case two cards would not add up to 14— which is the requirement for the two hearts.

b) If the sum of the evens is 14, the odds would be 5 and 9 and the evens would be 2,4,8. This can happen if we put the hearts as 5 and 9 and 2 goes to spades, and since the total of the blacks is 10, clubs must be the 8 of clubs and 4 would be the 4 of diamonds.

c) If the sum of the evens is 16, the evens should be 2, 4 and 10; the odds must be 7 and 9. But this is not possible because if we take the two hearts to total 14, it is not possible.

Thus, the solution is: 2 of spades, 5 and 9 of hearts, 4 of diamonds, and 8 of clubs

Option (d) is correct.

CHAPTER 6

Sequence and Series

Sequences and series (as the name suggests) consists of questions where you are supposed to understand the logic behind a given sequence or series of number/alphabets. Based on this understanding you are supposed to determine either:

(a) A continuation to the series

(b) The immediate next term of the series

or

(c) A missing term/terms within the series

While preparing yourself to solve such questions you should improve your ability to spot a particular relationship between terms within the series. The common logical premises used for questions are given here.

SERIES BASED ON NUMBERS

1. Squares
2. Squares + something or squares – something e.g. next term in the series
   24, 35, 48, 63, 80... will be 99, since the series is $(5^2 + 1)$, $(6^2 + 1)$, etc.
3. Cubes, cubes + something, cubes – something
4. Geometric series
5. Arithmetic series
6. Harmonic series
7. Series of constantly changing additions or multiplications, e.g.,
   (i) 2, 6, 24, 120,..... (The terms are got by ×3, ×4, ×5, and so on.)
   (ii) 11, 16, 22, 29, 37,..... (The series follows a logic of +5, +6, +7, +8 and so on.)
8. Binary numbers systems or number systems with other bases like Base 3, Base 4 etc.
9. Intermingled series–
   e.g., 2, 5, 6, 10, 18, 15, 54
   In this series alternate terms starting with the first form a GP with common ratio 3, while the 2^{nd}, 4^{th}, 6^{th} terms form an AP 5, 10, 15.....

SERIES BASED ON ALPHABETS

In such series, the most crucial aspect is to know the position of each alphabet in the alphabet series A to Z, both from start to end.

Thus, the following reference numbers for each alphabet become important:

Going Forward	1	2	3	4	5	6	7	8	9	10	11	12	**13**
Alphabet	A	B	C	D	E	F	G	H	I	J	K	L	**M**
Going Back from Z	26	25	24	23	22	21	20	19	18	17	16	15	**14**

Going Forward	**14**	15	16	17	18	19	20	21	22	23	24	25	26
Alphabet	**N**	O	P	Q	R	S	T	U	V	W	X	Y	Z
Going back from Z	**13**	12	11	10	9	8	7	6	5	4	3	2	1

Notice that M and N are the two middle points of this series. Let us now look at a few solved examples before we go into the practice exercise.

Illustration 1

Which of the numbers given below would come next in the series of numbers?

1, 9, 25, __

(a) 36 (b) 25

(c) 49 (d) None of these

➢ **SOLUTION**

The first number is 1, second is 9 and third is 25. We can easily observe that the numbers in the series are squares of consecutive odd numbers. Hence, the next number would be the square of 7, which would be 49. Thus option (c) is correct.

Illustration 2

Which of the following letters would come next in the series of letters?
Z, W, R, K, __

(a) A (b) C
(c) B (d) D

SOLUTION

In the given series from the end of the alphabet series, Z is the 1st alphabet, W is the 4th alphabet, R is the 9th alphabet and K is the 16th alphabet. Thus, the next letter in the series will be the 25th alphabet from the end or the second from the start. Hence, the next alphabet must be B. Thus, (c) is the correct option.

Illustration 3

What would come next in the following series of numbers?
1, 2, 10, 37, __

(a) 62 (b) 91
(c) 101 (d) None of these

SOLUTION

The series is following the +1, +8, +27 routine. Hence, the next number must be at an interval of +64 and should be 101. Thus option (c) is the correct answer.

Illustration 4

What would come next in the following series?
A_{10}, D_{15}, G_{20}, J_{25} __

(a) N_{18} (b) O_{20}
(c) M_{16} (d) None of these

SOLUTION

In the series of letters represented above there is a gap of two letters between every two consecutive terms and hence, the next letter will be M (after skipping K and L). Thus, the correct answer should be M_{30}. Thus option (d) is correct.

Illustration 5

What should come next in the series below?
122343456_____

(a) 45678 (b) 6789
(c) 34567 (d) None of these

SOLUTION

We can see that the series is a group of joined series as 12, 234, 3456, and hence the next terms should be 45678. Thus option (a) is correct.

CHAPTER 7

Blood Relations

Questions on blood relations are common under the logical reasoning section of aptitude exams and have also been asked regularly in past UPSC papers as well as at most state public service examinations. As the name suggests, questions on blood relations involve working out the familial relationships between people in a family.

There are essentially two kinds of questions that can be classified under this area: *viz*

(A) Puzzle-type questions with a family relationship component

(B) Statement-based relationship questions

Let us look at these one by one:

(A) Puzzle-type questions: This question type normally involves situations where there is some family relationship that is part of the puzzle. The following illustrations would make it clear to you:

Illustration 1

In a family of six persons—Abhay, Banta, Cathy, Deepak, Emily and Fatima, there are three males and three females. There are two married couples and two persons are unmarried. Each one of them likes different cold drinks.

Emily who likes Coke, is the mother-in-law of Abhay, who is the wife of Cathy. Deepak is the father of Fatima and he does not like Thumps Up or Pepsi; Banta likes Limca and is the sister of Fatima who likes Sprite. Cathy does not like Pepsi. Fanta is another cold drink.

1. Who among the following likes Thumps Up?
   (a) Cathy (b) Deepak
   (c) Abhay (d) Data inadequate
2. How is Fatima related to Emily?
   (a) Brother (b) Son
   (c) Father (d) Daughter
3. One of the married couple is:
   (a) Deepak–Banta (b) Deepak–Emily
   (c) Banta–Fatima (d) Emily–Fatima
4. Which of the following cold drinks is liked by Abhay?
   (a) Thums Up (b) Fanta
   (c) Pepsi (d) Data inadequate
5. How many sons does Emily have?
   (a) Four (b) Three
   (c) Two (d) One

Illustration 2

In a family gathering, there are two males who are grandfathers and four males who are fathers. In the same gathering there are two females who are grandmothers and four females who are mothers. There is at least one grandson or a granddaughter present in this gathering. There are two husband-wife pairs in this group. These can either be a grandfather and a grandmother, or a father and a mother. The single grandfather (whose wife is not present) has two grandsons and a son present. The single grandmother (whose husband is not present) has two granddaughters present. A grandfather or a grandmother present with their spouses does not have any grand son or grand daughter present.

1. What is the minimum number of people present in this gathering?
   (a) 10 (b) 12
   (c) 14 (d) 16

Illustration 3

Direction for Questions 1–3: Read the following information carefully and answer the questions given

In a joint family of seven persons L, M, N, O, P, Q and R two are married couples.

"R" is a housewife and her husband is a lawyer. "N" is the wife of "M", "L" is an engineer and is the granddaughter of "R", and "O" is the father-in-law of "N", a doctor, and father of "P", a professor. "Q" is L's brother and M's son.

1. How is P related to M?
   (a) Son (b) Brother
   (c) Daughter (d) Data inadequate

2. How is Q related to O?
   (a) Grand father (b) Uncle
   (c) Grand son (d) Data inadequate
3. Who is M's father?
   (a) O (b) R
   (c) N (d) Data inadequate

Illustration 4

Direction for Questions 1–4: Answer the questions based on the following information.

Hello, Ciao and Vanakkam are related to each other.

I. Among the three, one is Hello's legal spouse, another is Ciao's sibling and the third is Vanakkam's sister-in-law.

II. Hello's legal spouse and Ciao's sibling are of the same sex.

(Assume: There are no same-sex marriages.)

1. Who is the married man?
   (a) Hello (b) Ciao
   (c) Vanakkam (d) Cannot be determined
2. Who is the married woman?
   (a) Hello (b) Ciao
   (c) Vanakkam (d) Cannot be determined
3. The number of males amongst the group is
   I. 1 II. 2
   (a) I (b) II
   (c) I or II (d) Neither I nor II
4. Who is Ciao's sibling?
   (a) Hello (b) Ciao
   (c) Vanakkam (d) Cannot be determined

SOLUTIONS

Illustration 1

While solving logical reasoning questions always try to place the direct information first and keep any indirect clues aside for later use.

In this question you will find that:

Emily likes Coke .....→ Direct clue

Deepak is the father of Fatima → Direct clue

Deepak does not like Thums up or Pepsi → Indirect clue,.... and so on.

Keep in mind the following points while solving all questions on logical reasoning:

1. First use the direct clues.
2. In the case of a family tree the diagram should essentially be a multilevel diagram to ensure clarity of being able to see multiple generations on the same diagram.
3. Males and females should be marked separately with some symbols. One suggestion is the females can be marked inside a circle/bracket or with an underlined letter as A or alternately males can be marked with a + sign and females with a – sign.
4. Relationships should be marked using horizontal links or vertical links.

The following diagrams will emerge from the clues–

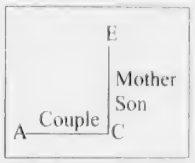


Figure 1

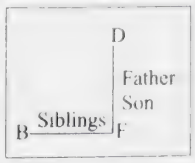
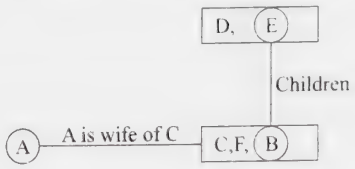


Figure 2

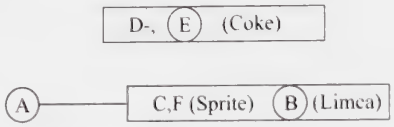
At this stage, you know that A, B and E are the three females and C, D and F are the three males; you also know that A and C are one of the two couples.

Hence, the other couple must be D and E. (They must be married to each other since they have children.)

This leads to a combined diagram which looks as below:



To this information, add the likes and dislikes of individuals as follows:



Further D does not like Thums up or Pepsi, hence he must like Fanta; Cathy does not like Pepsi (hence Thums Up) and Abhay likes Pepsi.

The answers are:

1. Cathy
2. Fatima is Emily's son
3. Deepak–Emily is a married couple
4. Pepsi
5. Two sons (C and F)

Illustration 2

The family needs to have a minimum of 12 people.

GF1-GM1, GF2, GM2

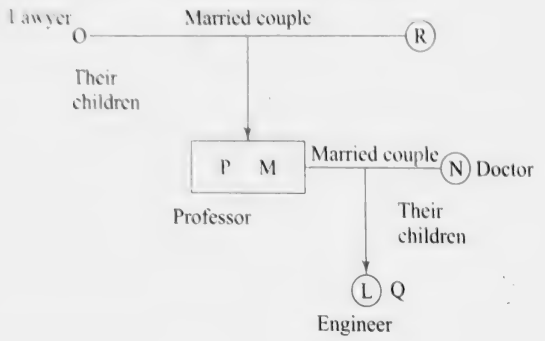
F1-M1, F2, M2

Generation 1	Grandfather 1 and his wife grandmother 1	Single grandfather 1	Single grandmother 1
Generation 2	Father-Mother (husband-wife pair)	Father	
Generation 3		Two boys	Two girls

The above table gives us a total of 11 people who are compulsorily present. We need to add 1 more female (who is a mother) to this group, as there are 4 mothers in the gathering.
Thus a total of 12 people.
Option b is correct.

Illustration 3

The following family relationships emerge



The answers are:
7. Option (b) (Brother)
8. Option (c) (Grandson)
9. O is M's father. Option (a) is correct.

Illustration 4

From the statement that Ciao's sibling and Hello's legal spouse are of the same sex, we can conclude that Hello cannot be the sibling of Ciao (because Hello's sex would be different from his/her legal spouse's sibling). Hence, Ciao's sibling must be Vanakkam.

This gives us four basic possibilities for the husband–wife pair:

(1) Hello (male) – Vanakkam (female; Hello's wife as well as Ciao's sibling); Ciao (Vanakkam's sibling; sex not known). In this case, the situation does not allow for Vanakkam's sister-in-law. Hence, we can reject this as a possible solution to the situation.

(2) Hello (female) – Vanakkam (male; Hello's husband as well as Ciao's sibling); Ciao (Vanakkam's sibling – sex not known). In this case again, the situation does not allow for Vanakkam's sister-in-law. Hence, we can reject this as a possible solution to the situation.

(3) Hello (male) – Ciao (female; Hello's wife as well as Vanakkam's sibling); Vanakkam (Ciao's sibling – female as she has to be the same sex as Hello's spouse). In this case, again the situation does not allow for Vanakkam's sister-in-law. Hence, we can reject this as a possible solution to the situation.

(4) Hello (female) – Ciao (male; Hello's husband as well as Vanakkam's sibling); Vanakkam (Ciao's sibling – male as she has to be the same sex as Hello's spouse). In this case, the last condition of Vanakkam's sister-in-law is met because Hello is the sister-in-law of Vanakkam. Thus, we conclude that this is the only possible solution given the clues. The answers then become:

1. Ciao is the married man—Option b.
2. Hello is the married woman—Option a.
3. There are exactly two males—Ciao and Vanakkam—Option b.
4. Vanakkam—Option c.

(B) Statement-based relationship finding questions:
In this question type we typically have to determine the relationship between two individuals whose relationship is mentioned in a roundabout manner.
In such questions, it is advisable to look at a problem from one's own perspective i.e., if you place yourself as the central figure in the relationship matrix, the solving of the question becomes much easier as the solution anchors itself to pre-existing relationships in your own mind. The following examples would make things clear to you.

EXAMPLE 1

1. Introducing a boy, Ralf said, "His mother is the only daughter of my mother-in-law." How is Ralf related to the boy?
   (a) Uncle (b) Father
   (c) Brother (d) Husband

EXAMPLE 2

2. Pointing to a man in a photograph, a woman said, "His brother's father is the only son (and child) of my grandfather." How is the woman related to the man in the photograph?
   (a) Mother (b) Sister
   (c) Aunt (d) Daughter

EXAMPLE 3

3. If Nina says, "Anna's father Rick is the only son of my father-in-law Mick". Then how is Bridgette, who is the sister of Anna, related to Mick?
   (a) Daughter (b) Wife
   (c) Daughter-in-law (d) None of these

EXAMPLE 4

4. 'X + Y' means 'X is the father of Y', 'X – Y' means 'X is the wife of Y', 'X × Y' means 'X is the brother of Y', 'X ÷ Y' means ,'X is the mother of Y' and 'X = Y' means 'X is the sister of Y'. On the basis of this information, what does X + Y – Z mean?
   (a) X is the father-in-law of Z
   (b) Z is the father of X
   (c) Z is the neice of X
   (d) X is the brother of Z

SOLUTIONS

Example 1: Mother-in-law's only daughter means wife. Thus the statement could be read as "His mother is my wife". Thus, Ralf must be the father of the boy.

Example 2: Read this as "his father" is my grandfather's only child. Obviously this means that the woman would be the sister of the man in the photograph. Option b is correct.

Example 3: From the statement, it is clear that Mick is Rick's father and Rick is Anna's father. Bridgette and Anna would thus be granddaughters of Mick. Option 4 is correct.

Example 4: X + Y means that X is the father of Y and Y – Z means that Y is the wife of Z. Thus, X is the father-in-law of Z. Option 1 is correct.

The key skills involved in solving questions on blood relations include:

(i) The ability to structure the family tree in terms of visualising the number of generations, the number of people in each generation, etc.

(ii) The ability to understand and use vital clues leading to information such as: husband–wife relationships, father-son relationships, mother–son relationships, sibling relationships.

(iii) The ability to keep count of the number of people being talked about in the problem.

(iv) The ability to convert clues written in language form into visual cues so that you do not need to read the text again and again. Also, converting the language clues to visual cues is critical for the purpose of being able to 'see' all the clues at one go.

CHAPTER 8

Direction Test

Questions on directions are standard ones that are asked in most aptitude exams, hence are a common component in the study of reasoning. As the name itself suggests, questions about directions would involve reasoning based on the eight directions on a map – viz: north, northeast, east, southeast, south, southwest, west and northwest. This question type requires a student to visualise these directions and the movement of an individual/relative positioning of places on a map.

The key skills involved in solving questions based on directions are the following:

(i) The ability to understand and interpret the clues written in plain language in terms of what it means direction wise

(ii) The ability to order the clues in the correct order of usage (as explained in the theory of logical reasoning)

(iii) The ability to understand basic mapping concepts like what are the basic directions, what direction one would start facing if one turns right while going north or for that matter even what direction one would be facing if one turns 45° right while moving southwards, etc.

(iv) The ability to create a picture to represent the flow as mentioned in the problem.

Illustrated below are the solutions to a few typical questions on directions. We would urge you to first have a look at the questions and try to solve the same on your own before looking at the solutions provided.

EXAMPLE 1

Direction for Questions 1–3: Answer the questions based on the following information.

The city K is 30 km to the southeast of Z, while Y is 50 km to the northwest of K. Also, H is 38 km to the southeast of Y. L lies in the direct route between Y and K, and its distance from H is 14 km. G also lies on this route and is exactly midway between L and Y.

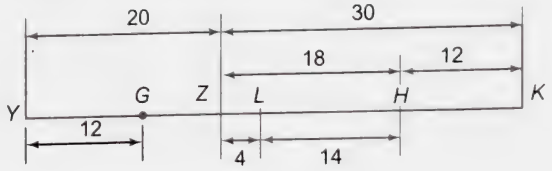
1. A car starting from K at 9 am and running at a constant speed towards Y reaches H at 9.24 am and then reaches G at
   (a) 9.18 am (b) 10.16 am
   (c) 10.36 am (d) 10.42 am
2. If M is 1 km to the southeast of L, then it is exactly midway between
   (a) H and L (b) Y and K
   (c) H and Z (d) None of these
3. The distance from G to H is
   (a) 26 km (b) 24 km
   (c) 12 km (d) 16 km

SOLUTIONS TO EXAMPLE 1

The figure below shows the respective positioning of the cities K, H, L, Z, G and Y.

From the figure we can deduce the answers using the following thinking:

(1) The car covers K to H—a distance of 12 kms in 24 minutes according to the figure (starting at 9 am and reaching at 9:24 am). From H to G, the distance is 26 kms. (18 km to Z + 8 km to G). Since the car has



covered 12 kms in 24 minutes, it is obvious that it is taking 2 minutes to cover 1 km. We also know that the speed of the car is constant throughout. Hence, the car would take another 52 minutes to cover the 26 km distance between H and G. This means that the car would reach G at 10:16 am (52 minutes after 9:24 am). Hence, option (b) is correct.

(2) The total distance between Y and K is 50 km (20 km from Y to Z and 30 km from Z to K). If M is 1 km southeast of L, it means that M is 4+1 = 5 km from Z. This would also mean that M is 25 km from Y. Hence, M would be exactly midway between Y and K. Option (b) is correct.

(3) G to Z, the distance is 8 km (20 km from Y to Z – 12 km from Y to G). Further, from Z to H the distance

is 18 km (from the figure based on the logic that if H is 38 km southeast of Y and Z is 20 km southeast of Y, the distance from Z to H would be 38 – 20).
Total distance would be 8 + 18 = 26 km. Option (a) is correct.

EXAMPLE 2

Direction for Questions 4–6: Refer to the table given below. There are six cities viz., Amsterdam, Bhubaneshwar, Calgary, Denver, El Dorado and Frankfurt. Their positions with respect to one another on a map are described through the five clues given below.
Calgary is to the south of Frankfurt, but to the west of Denver.
Frankfurt is to the south of El Dorado, which is east of Denver.
Amsterdam is south of Bhubaneshwar, which is west of Frankfurt.
El Dorado is south of Amsterdam, which is west of Calgary.
Denver is south of Frankfurt, which is west of Amsterdam.

4. Which of the following are situated to the northeast of at least one other city?
   (a) Bhubaneshwar, Amsterdam and El Dorado
   (b) Amsterdam and El Dorado
   (c) Bhubaneshwar, Amsterdam
   (d) Amsterdam, Frankfurt and Denver
5. Which of the following are to the northeast of Frankfurt?
   (A) Amsterdam (B) El Dorado (C) Calgary
   (a) A only (b) B only
   (c) A and C (d) A and B
6. Which of the following statements cannot be derived from the given information?
   (a) Bhubaneshwar is to the west of Amsterdam.
   (b) Denver is to the south of Amsterdam.
   (c) Denver is to the south of Calgary.
   (d) Amsterdam is to the west of El Dorado.

SOLUTIONS TO EXAMPLE 2

It can be concluded that the northsouth distribution of the cities would be one of the following:

Possibility 1	Possibility 2
Bhubaneshwar	Bhubaneshwar
Amsterdam	Amsterdam
El Dorado	El Dorado
Frankfurt	Frankfurt
Denver	Calgary
Calgary	Denver

The eastwest placement would be as follows:

Bhubaneshwar	Frank-furt	Amster-dam	Calgary	Denver	El Dorado

The grid possibilities based on these two arrangements would be as follows:

Possibility 1:

Bhubanesh-war					
		Amster-dam			
					El Dorado
	Frank-furt				
				Denver	
			Calgary		

Possibility 2:

Bhubanesh-war					
		Amster-dam			
					El Dorado
	Frank-furt				
			Calgary		
				Denver	

Based on these we can answer the questions as follows:

4. Looking at the above tabular picture of the placement of the cities as northsouth or eastwest, we can see that El Dorado and Amsterdam are the only cities that satisfy this condition. Denver and Calgary could be northeast to each other depending on which possibility we consider—in Possibility 1 Calgary is not northeast of any city and hence we can eliminate the possibility of Calgary being northeast of at least one other city. Similarly on the basis of Bhubaneshwar being west most cannot be to the northeast of any city (as Denver would. Option (2) is correct.
5. Both A and B. Hence, option (d) is correct.
6. Option (c) cannot be concluded from the information as we cannot decide on the northsouth placement between Calgary and Denver.

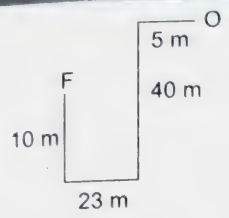
EXAMPLE 3

1. Jack runs 10 m south from his flat, turns left and walks 23 m, again turns left and walks 40 m, then turns right

and walks 5 m to reach his office. In which direction is the office from his house?

(a) East (b) Northeast
(c) Southwest (d) North

SOLUTIONS TO EXAMPLE 3



When we will look from F (Flat) to O (office) in the figure, it can be observed that the direction is northeast. Option (b) is correct.

EXAMPLE 4

2. I am facing north. I turn 90° in the clockwise direction and then 135° in the same direction and then 270° anti-clockwise. Which direction am I facing now?

(a) Southwest (b) South
(c) West (d) Northwest

SOLUTIONS TO EXAMPLE 4

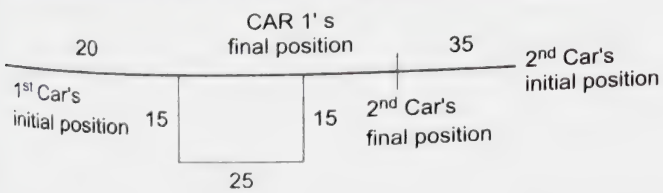
After turning 90° I am looking in the east direction, then after turning 135° in the same direction I would be facing the southwest direction. At last after turning 270° anticlockwise I would be facing the northwest direction. Option (d) is correct.

EXAMPLE 5

Two cars start from the opposite points of straight part of the National Higway 8, 100 km apart. The first car runs for 20 km. It then takes a detour—takes a right turn, goes straight for 15 km. It then turns left, runs for another 25 km and then takes the straight connecting road to reach back on the main highway. In the meantime, due to a minor breakdown, the other car has run only 35 km along the main highway. What would be the distance between the two cars at this point?

(a) 20 km (b) 30 km
(c) 45 km (d) 10 km

SOLUTIONS TO EXAMPLE 5



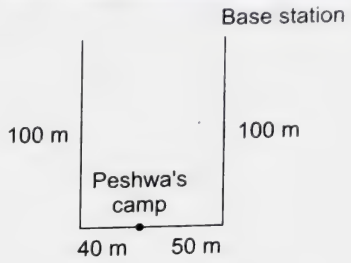
From the above figure it is clear that Car 1 would move 45 kms along the highway, while Car 2 would run 35 kms along the same highway. Naturally, the distance between the two cars at this time would be 20 km. Option (a) is correct.

EXAMPLE 6

A messenger was returning to his base station, which was in front of him to the north. When his base station was 100 m away from him, he turned to the left and moved 50 m to deliver the last message to the Peshwa's troops. He then moved in the same direction for 40 m, turned to his right and moved 100 m. How many m away he was now from his base station?

(a) 0 (b) 150
(c) 90 (d) 100

SOLUTIONS TO EXAMPLE 6



Option (c) is correct.

EXERCISE

Direction for Questions 1–4: Answer the questions independent of each other.

1. A watch in Huckleberry Finn's house reads 4·30. If the minute hand points towards the west, in which direction does the hour hand points?
   (a) Southward (b) Northeast
   (c) Northwest (d) North
2. Hanuman, while looking for the *Sanjeevani booty* travels 3 km to the west, turns left and goes 3 km, turns right and goes 1 km, again turns right and goes 3 km. How far is he from the starting point?
   (a) 7 km (b) 6 km
   (c) 5 km (d) 4 km
3. Raveena walks 10 km south from her house, turns left and walks 25 km, again turns left and walks 40 km, then turns right and walks 5 km to reach her office. In which direction was the office from her house?
   (a) Southwest (b) Northeast
   (c) East (d) North

   For the above question, what is the distance of her office from her home?
   (a) 30 (b) $30 \times 2^{1/2}$
   (c) $45 \times 2^{1/2}$ (d) None of these

Directions for Questions 5–9: These questions are based on the diagram given below showing four persons Amar, Bhushan, Chandu and Dilip stationed at the four corners of a square piece of plot as shown.

CHAPTER 9

Puzzle Test

Reasoning puzzles are a favourite question type in all aptitude examinations. In the scheme of chapters on reasoning contained in this book, we have created separate chapters for specific kinds of puzzles, which are often asked (like arrangements, rankings etc., which constituted the chapters prior to this one).

All other categories of puzzles—which cannot be specifically categorised as any of the foregoing chapters—you will get to see and practice in this chapter.

As the name suggests, questions on puzzles challenge you to match multiple factors (like name, colour of shirt, place of living, car model driven etc.).The key skills involved in solving questions on puzzles include but are not limited to:

(i) **The ability to make a relevant tabular structure for using the clues seamlessly:** For example, suppose you have five people A, B, C, D, E wearing five colour of shirts red, yellow, green, blue and white drinking five kinds of soft drinks Coke, Pepsi, Mirinda, Thums Up and Seven Up—the solution table structure would look like this:

	Red	Yellow	Green	Blue	White	Coke	Pepsi	Mirinda	Thums Up	Seven Up
A										
B										
C										
D										
E										
Coke										
Pepsi										
Mirinda										
Thums UP										
Seven Up										

In the table above, you can see clearly that there is a direct correlation structure between each of the three "variables" in the problem. More of this you would get to see and experience as you move through the solved illustrations and the exercises that follow it.

(ii) **The ability to order the clues in the correct order of usage (as explained in the theory of logical reasoning):** This includes the ability to perceive the direct clues and use them first to set up the 'framework' of problem solving.

(iii) **The ability to perceive what indirect clues are talking about – and how to use them;**

(iv) **The ability to convert clues written in language form into visual clues so that you do not need to read the text again and again.** Also, converting the language clues to visual cues is critical for the purpose of being able to "see" all the clues at one go.

Illustrated below are the solutions to a few typical questions on puzzles. We would advise you to first have a look at the questions and try to solve the same on your own before looking at the step-by-step process of solving the same.

ILLUSTRATION 1

At a fancy dress party people were asked to dress as an object that represented their professions.

Mr. Abhijit, Mrs. Banerjee, Mrs. Chatterjee, Mr. Dipanjan De and Mr. Elangovan were among the guests. The costumes included a leaf, a pen, a fork, a camera reel, and a stethoscope. The professions included a photographer, a gardener, a compounder, a teacher, and a cook.

- Mr. Abhijit is a teacher.
- Neither Mrs. Banerjee nor Mrs. Chatterjee was dressed as a fork.
- None of the men is a compounder.
- Mr. Dipanjan De is dressed as a camera reel.
- Mrs. Chatterjee is a gardener.

1. Which person is dressed as a stethoscope?
   (a) Abhijit (b) Banerjee
   (c) Chatterjee (d) Dipanjan
2. What is Elangovan's profession?
   (a) Cook (b) Gardener
   (c) Compounder (d) Teacher

SOLUTION

Putting the direct Clues 1 and 4 in the table we get:

Male	Female	Female	Male	Male
Abhijit	Banerjee	Chatterjee	Dipanjan De	Elangovan
Pen			Camera reel	
Teacher			Photographer	
From clue 1			From clue 4	

At this point if we use clue 2, it is evident that it must be Elangovan who is dressed as a fork. Also using clue 5 (Mrs. Chatterjee is a gardener) completes the solution.

Male	Female	Female	Male	Male
Abhijit	Banerjee	Chatterjee	Dipanjan De	Elangovan
Pen		Leaf	Camera reel	Cook
Teacher		Gardener	Photographer	Fork
		From Clue 5		From clue 2 once we know the details of Dipanjan and Abhijit.

Obviously, Mrs. Banerjee would be the compounder.
The solutions are: 1. Banerjee (option b)
2. Elangovan is the cook. Option (a) is correct.

ILLUSTRATION 2

Directions for questions 12 to 15: Answer the following questions based on the information below:
For admission into vacant seats of higher secondary classes, most schools prescribe an admission test. There is no uniformity in the nature of these tests, however. Anuja Sinha, Amit Agnihotri, Himesh Reshammiya, Mahima Sharma and Sonit Kala, whose parents had been transferred to Lucknow this summer and who were all seeking admission into Class 9, appeared in entrance tests of different schools. The number of questions in all these tests happened to be different. Amit wrote an entrance test containing exactly twice as many questions as the test that Himesh wrote, Sonit wrote a test that contained 160 questions, but it was neither for admission into City Montessori School's nor for Scindia School. Delhi Public School test contained 200 questions, but neither Himesh nor Mahima wrote this test. City Montessori School test had 120 questions, but it was not written by Himesh. The test that Mahima Sharma wrote had 25% less questions than the test Amit Agnihotri wrote but it was not for City Montessori School and Scindia School.
Anuja did not write Millennium School's or Saint Mary's Convent's test. When all the tests are compared with regard to number of questions in them, St. Mary's Convent falls exactly in the middle of the list. Each of the five students wrote exactly one of the tests – the longest one of which contained 200 questions. There is no penalty for wrong answers.

3. Anuja Sinha wrote the test for admission into which school?
   (1) Scindia
   (2) Sanskar Valley
   (3) City Montessori School
   (4) Delhi Public School
4. If Amit Agnihotri secured only 50 percent marks in the test, each question carrying one mark, how many marks did he secure in this test?
   (1) 50 (2) 60
   (3) 100 (4) None of these
5. If both Sonit and Amit secured equal marks in their respective tests and the test Sonit wrote carried 5 marks per question, how many marks did each question in Amit's test carry? It is given that both Amit and Sonit answered 75% questions of their respective tests correctly. Wrong answers did not fetch any marks.
   (1) 1 (2) 2.5
   (3) 3 (4) 4
6. Which of the following combinations is true?
   1. Anjua Sinha – City Montessori School– 180 Questions
   2. Amit Agnihotri – Millennium School– 160 Questions
   3. Himesh Reshammiya – Scindia School– 100 Questions
   4. Mahima Sharma – Scindia School– 150 Questions

Solution: Reaction Tracker

With respect to the number of questions in each test it can be inferred that the number of questions are 200,120,160. Two values are unknown and to compensate for them we have been given that the number of questions in the tests of Amit Agnihotri would be (2x), Himesh Reshammiya (x) and Mahima Sharma (1.5x). The starting table for this situation would be:

	Anuja Sinha	Amit Agnihotri (2x)	Himesh R (x)	Mahima Sharma (1.5 x)	Sonit Kapoor	CMS	Scindia	Millenium	St. Marys	DPS
200			NO	NO	NO	NO	NO	NO	NO	YES
160	NO	NO	NO	NO	YES	NO	NO			NO
120			NO		NO	YES	NO	NO	NO	NO
??					NO	NO				NO
??					NO	NO				NO
CMS			NO	NO	NO					
Scindia				NO	NO					
Millennium	NO									
St. Mary's	NO									
DPS			NO	NO	NO					

From the table it is clear that DPS (200) and CMS (120) are shared between Anuja and Amit. This means two possibilities – Anuja 120 and Amit 200, or Amit 120 and Anuja 200. If we take the second possibility into account things would not work out because:

If Amit is 120, the missing numbers are 90 (25% less for Mahima) and 60 (Amit's test had twice Himesh's test). Then the number of questions placed in ascending order would be: 60, 90, 120 (CMS), 160, 200 (DPS). This situation contradicts the condition that when all the tests are compared as to number of questions in them, St. Mary's Convent falls exactly in the middle of the list.

Thus, we must consider the other option i.e., Amit had 200 questions and Anuja 120. Then the number of questions is 200, 160, 150 (Mahima), 120, 100. The table would now look as follows:

	Anuja Sinha	Amit Agnihotri (2x)	Himesh R (x)	Mahima Sharma (1.5 x)	Sonit Kapoor	CMS	Scindia	Mille-nium	St. Mary's	DPS
200	NO	YES	NO	NO	NO	NO	NO	NO	NO	YES
160	NO	NO	NO	NO	YES	NO	NO	YES	NO	NO
120	YES	NO	NO	NO	NO	YES	NO	NO	NO	NO
150	NO	NO	NO	YES	NO	NO	NO	NO	YES	NO
100	NO	NO	YES	NO	NO	NO	YES	NO	NO	NO
CMS	YES	NO	NO	NO	NO					
Scindia	NO	NO	YES	NO	NO					
Millennium	NO	NO	NO	NO	YES					
St. Mary's	NO	NO	NO	YES	NO					
DPS	NO	YES	NO	NO	NO					

The solutions are:

3. CMS. Option (c) is correct.
4. He would score 100 marks. Option (c) is correct.
5. Sonit and Amit would score 600 marks out of 800 (The total marks would be 800 because Sonit's test has 160 questions of 5 mark each totaling 800). Hence, Amit's score would also be out of 800 marks—4 marks per question. Option (d) is correct.
6. Option (c) is correct.

ILLUSTRATION 3

Directions for Question 7: Read the information and answer the question.

Four engineers, designated as CE, SE, EE and AE, read a certain number of newspapers early in the morning. One of them reads four newspapers, another reads three newspapers, the third reads two newspapers while the fourth one reads one newspaper. Below are some additional facts regarding the names of these officers:

i. Nahiri is not the EE.
ii. Hari is the AE.

iii. Nahiri is not the CE and he reads more number of newspapers than Lahiri.
iv. The one who is the CE reads more number of newspapers than Lahiri.
v. The person who is the SE reads the maximum number of newspapers.
vi. Bahiri does not read two newspapers.

7. Which of the following statements is necessarily true?
   1. Hari is the AE and reads two newspapers.
   2. Lahiri is the EE and reads one newspaper.
   3. Bahiri is the CE and reads three newspapers.
   4. Nahiri is the EE and reads four newspapers.

SOLUTION

From statement (ii), Hari is AE. From statements (i) and (iii), Nahiri is not the CE or the EE. Hence Nahiri is the SE and reads four newspapers as from statement (v). From Statement (iv), Lahiri is not the CE; this obviously means that Lahiri is the EE and Bahiri is the CE. From statement (iv) and (vi), Bahiri reads three newspapers. As he reads more than at least one person, we cannot allocate one newspaper to him. Hence Bahiri must read three newspapers. Hence the final distribution is as follows:

Engineer	Name	Number of Newspapers
AE	Hari	2/1
CE	Bahiri	3
SE	Nahiri	4
EE	Lahiri	1/2

Hence, option (3) is the correct answer.

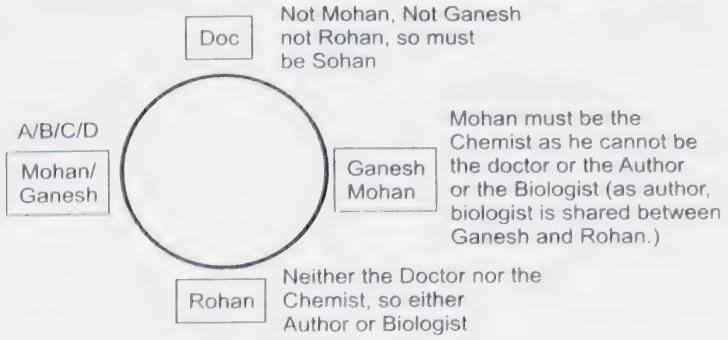
ILLUSTRATION 4

Four brothers Rohan, Sohan, Mohan and Ganesh are at their annual family property fight sitting across a circular table. Their occupations are— author, biologist, chemist and doctor, but not necessarily in that order. Ganesh starts by setting the agenda of the meeting and after him the doctor gives a long discourse of what is right and what is wrong. Rohan is sitting across the doctor and next to the chemist. Mohan is silent throughout the meeting and the chemist speaks only at the very end.

8. The profession of Rohan is
   (a) author (b) biologist
   (c) doctor (d) Cannot be determined
9. Who among the following is the chemist?
   (a) Sohan (b) Mohan
   (c) Ganesh (d) Cannot be determined

SOLUTION

The solution of the above question can be seen in the figure below. (Note the interpretation of the last statement – Mohan was silent throughout the meeting in that Mohan spoke only at the end.)



Based on the interpretations of the figure above we can answer the questions as:
8) Rohan could be the Author or the Biologist. Hence, we cannot determine his profession. Option (d) is correct.
9) Mohan was the Chemist. Option (b) is correct.

ILLUSTRATIONS 5 and 6

Directions for Questions 10 to 12: Study the following information carefully and answer the questions given below:
Four people were being interviewed for the same job. They were all interviewed on the same day, but in different rooms (Dharti, Prithvi, Akash and Agni), at different times and by different persons. Determine the name of each candidate, which room they were interviewed in and their appointment time, and answer the questions.

(i) Aishwarya's appointment was just after Mr. Sharma's, which was just after that of the person in room Prithvi.
(ii) Mr Narurkar's appointment was at least 2 hr later in the day than Bhagat's.
(iii) Mr Joshi's appointment was just after the person who had an interview in room Agni, who had an appointment just after Chiranjeev.
(iv) Three of the four interviewees were: Dhanush, the one with the interview in room Dharti, and the person who had an appointment at 1 pm.
(v) The four people were interviewed at 11 am, 12 noon, 1 pm and 2 pm.
(vi) Joshi, Narurkar, Zaidi and Sharma were the interviewers, whereas Aishwarya, Bhagat, Chiranjeev and Dhanush were the interviewees.

10. Sharma's appointment was with
    (1) Aishwarya
    (2) Bhagat
    (3) Chiranjeev
    (4) Cannot be determined

CHAPTER 10

Set Theory and Venn Diagrams

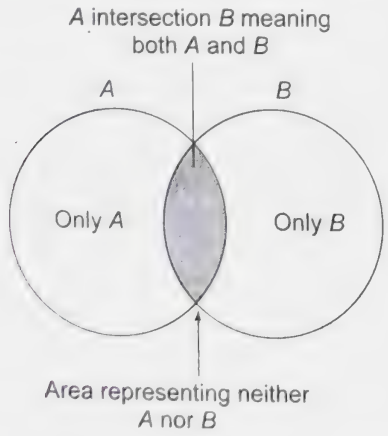
Questions on Set theory/Venn Diagrams are one of the most commonly asked questions in CAT. Besides, they are also very critical for all other aptitude exams - like XAT, IIFT, SNAP, NMAT, UPSC-CSAT, IPMAT, Banking etc.

Let us first take a look at some standard theoretical inputs related to set theory.

SET THEORY

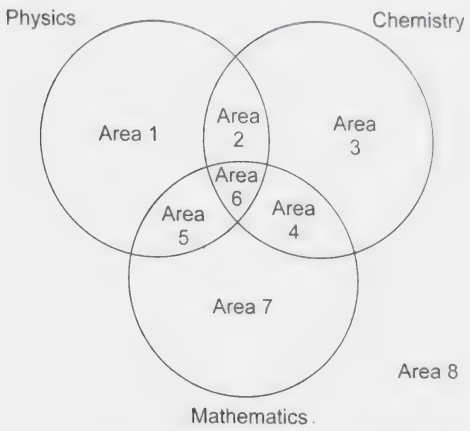
Look at the following diagrams:

Figure 1: Refers to the situation where there are two attributes A and B. (Let's say A refers to people who passed in Physics and B refers to people who passed in Chemistry.) Then the shaded area shows the people who passed both Physics and Chemistry.



In mathematical terms, the situation is represented as:
Total number of people who passed at least 1 subject = $A + B - A \cap B$

Figure 2: Refers to the situation where there are three attributes being measured. In the figure below, we are talking about people who passed Physics, Chemistry and/or Mathematics.



In the above figure, the following explain the respective areas:

Area 1: People who passed in Physics only.
Area 2: People who passed in Physics and Chemistry only (in other words—people who passed Physics and Chemistry but not Mathematics).
Area 3: People who passed Chemistry only.
Area 4: People who passed Chemistry and Mathematics only (also, can be described as people who passed Chemistry and Mathematics but not Physics).
Area 5: People who passed Physics and Mathematics only (also, can be described as people who passed Physics and Mathematics but not Chemistry).
Area 6: People who passed Physics, Chemistry and Mathematics.
Area 7: People who passed Mathematics only.
Area 8: People who passed in no subjects.

Also, take note of the following language which there is normally confusion about:

People passing Physics and Chemistry—represented by the sum of areas 2 and 6.

People passing Physics and Maths—represented by the sum of areas 5 and 6.

People passing Chemistry and Maths—represented by the sum of areas 4 and 6.

People passing Physics—represented by the sum of the areas 1, 2, 5 and 6.

In mathematical terms, this means:

Total number of people who passed at least 1 subject = P + C + M − P ∩ C − P ∩ M − C ∩ M + P ∩ C ∩ M

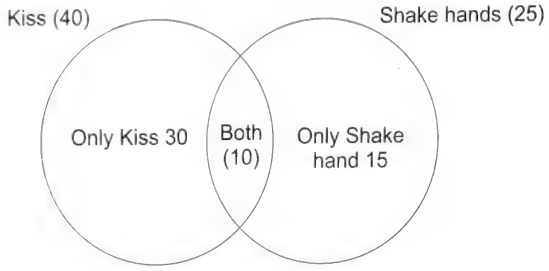
Let us consider the following questions and see how these figures work in terms of real time problem solving:

Illustration 1

At the birthday party of Sherry, a baby boy, 40 persons chose to kiss him and 25 chose to shake hands with him. 10 persons chose to both kiss him and shake hands with him. How many persons turned out at the party?

(a) 35 (b) 75
(c) 55 (d) 25

Solution



From the figure, it is clear that the number of people at the party were 30 + 10 + 15 = 55.

We can, of course, solve this mathematically as below:

Let n(A) = No. of persons who kissed Sherry = 40

n(B) = No. of persons who shake hands with Sherry = 25

and n(A ∩ B) = No. of persons who shook hands with Sherry and kissed him both = 10

Then using the formula, n(A ∪ B) = n(A) + n(B) − n(A ∩ B)

n(A ∪ B) = 40 + 25 − 10 = 55. Option (c) is the correct answer.

Illustration 2

Directions for Questions 1 to 4: Refer to the data below and answer the questions that follow:

In an examination 43% passed in Math, 52% passed in Physics and 52% passed in Chemistry. Only 8% students passed in all the three. 14% passed in Math and Physics and 21% passed in Math and Chemistry and 20% passed in Physics and Chemistry. Number of students who took the exam is 200.

Let set P, set C and set M denote the students who passed in Physics, Chemistry and Math respectively.

1. How many students passed in Math only?
   (a) 16 (b) 32
   (c) 48 (d) 80
2. Find the ratio of students passing in Math only to the students passing in Chemistry only?
   (a) 16:37 (b) 29:32
   (c) 16:19 (d) 31:49
3. What is the ratio of the number of students passing in Physics only to the students passing in either Physics or Chemistry or both?
   (a) 34/46 (b) 26/84
   (c) 49/32 (d) None of these
4. A student is declared pass in the exam only if he/she clears at least two subjects. The number of students who were declared passed in this exam is:
   (a) 33 (b) 66
   (c) 39 (d) 78

Solution Let P denote Physics, C denote Chemistry and M denote Maths.

% of students who passed in P and C only is given by % of students who passed in P and C − % of students who passed all three = 20% − 8% = 12%.

% of students who passed in P and M only is given by % of students who passed in P and M − % of students who passed all three = 14% − 8% = 6%.

% of students who passed in M and C only is:

% of students who passed in C and M − % of students who passed all three = 21% − 8% = 13%.

So, % of students who passed in P only is given by:

Total no. passing in P − No. passing in P and C only − No. passing P and M only − No. passing in all three →

52% − 12% − 6% − 8%− = 26%.

% of students who passed in M only is:

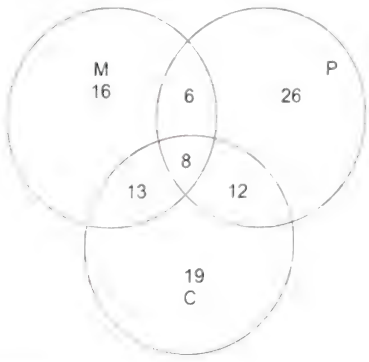
Total no. passing in M − No. passing in M and C only − No. passing P and M only − No. passing in all three →

43% − 13% − 6% − 8%− = 16%.

% of students who passed in Chemistry only is:

Total no. passing in C − No. passing in P and C only − No. passing C and M only − No. passing in all three →

52% − 12% − 13% − 8%− = 19%.



The answers are:

1. Only Math = 16% = 32 people. Option (b) is correct.
2. Ratio of Only Math to Only Chemistry = 16:19. Option (c) is correct.

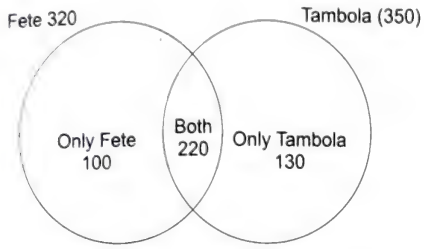
3. 26:84 is the required ratio. Option (b) is correct.
4. 39% or 78 people. Option (d) is correct.

Illustration 3

In the Mindworkzz club, all the members participate either in the Tambola or the Fete. 320 participate in the Fete, 350 participate in the Tambola and 220 participate in both. How many members does the club have?

(a) 410 (b) 550
(c) 440 (d) None of these

Solution



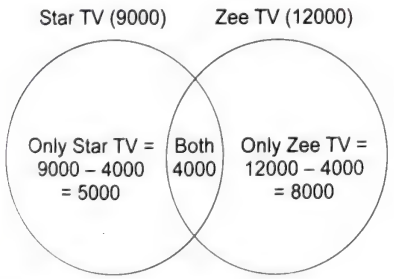
The total number of people = 100 + 220 + 130 = 450. Option (d) is correct.

Illustration 4

There are 20000 people living in Defence Colony, Gurgaon. Out of them 9000 subscribe to Star TV Network and 12000 to Zee TV Network. If 4000 subscribe to both, how many do not subscribe to any of the two?

(a) 3000 (b) 2000
(c) 1000 (d) 4000

Solution



The required answer would be 20000 – 5000 – 4000 – 8000 = 3000. Option (a) is the correct answer.

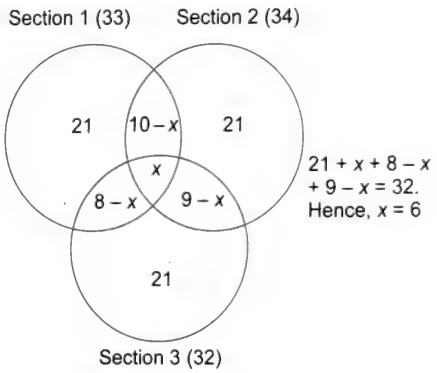
Illustration 5

Directions for Questions 1 to 3: Refer to the data below and answer the questions that follow:

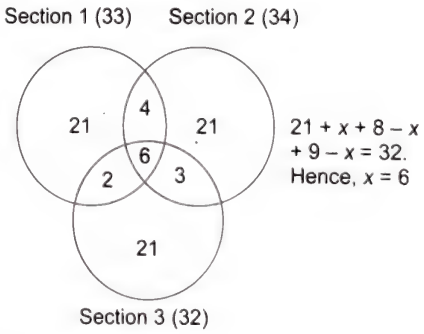
Last year, there were 3 sections in the Catalyst, a mock CAT paper. Out of them 33 students cleared the cut-off in Section 1, 34 students cleared the cut-off in Section 2 and 32 cleared the cut-off in Section 3. 10 students cleared the cut-off in Section 1 and Section 2, 9 cleared the cut-off in Section 2 and Section 3, 8 cleared the cut-off in Section 1 and Section 3. The number of people who cleared each section alone was equal and was 21 for each section.

1. How many cleared all the three sections?
   (a) 3 (b) 6
   (c) 5 (d) 7
2. How many cleared only one of the three sections?
   (a) 21 (b) 63
   (c) 42 (d) 52
3. The ratio of the number of students clearing the cut-off in one or more of the sections to the number of students clearing the cutoff in Section 1 alone is?
   (a) 78/21 (b) 3
   (c) 73/21 (d) None of these

Solution



Since, $x = 6$, the figure becomes:



The answers would be:

1. 6. Option (b) is correct.
2. 21 + 21 + 21= 63. Option (b) is correct.
3. (21 + 21 + 21 + 6 + 4 + 3 + 2)/21 = 78/21. Option (a) is correct.

CHAPTER 11

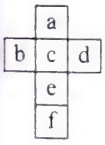
Cubes and Dices

Questions based on visualisation of cubes and dice are regularly asked in various Management entrance exams and also in various other aptitude exams. The key skills required, in order to solve these questions easily involve the ability to visualise the various shapes, their 3 – dimensional visualisation, as well as where the faces of the cube/dice would go, in case the object is cut open. Also, simultaneously you need to understand the thought process that would help you visualise how the faces of an open cube would appear when they are folded into a cube. The following visualisations should be known as a part of the theory of this chapter.

I. Visualising Opening and closing of Cubes:

For each of the figures given below of an open cube, the text accompanying the same describes which faces would be opposite each other when the cube is closed into a box.

Case 1:

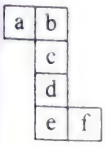


a lies opposite e

b lies opposite d

c lies opposite f

Case 2:

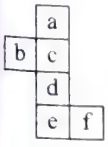


a lies opposite f

b lies opposite d

c lies opposite e

Case 3:

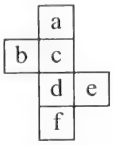


a lies opposite d

b lies opposite f

c lies opposite e

Case 4:

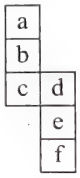


a lies opposite d

b lies opposite e

c lies opposite f

Case 5:



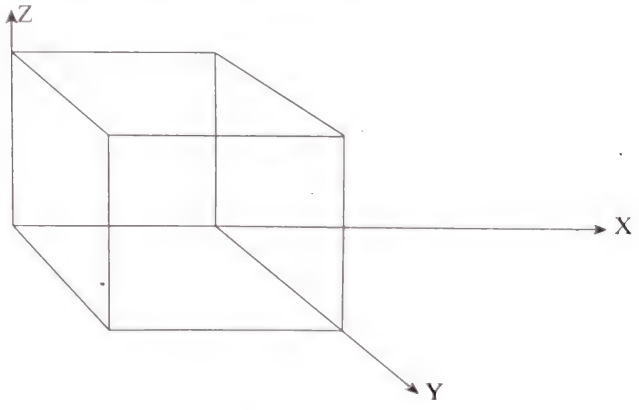
a lies opposite c

b lies opposite e

d lies opposite f

II. Cutting a Cube:

A cube has 6 faces, 12 edges and 8 vertices.



- When we make 'a' mutually parallel cuts along x – axis, 'b' mutually parallel cuts along y – axis and 'c' mutually parallel cuts along z axis, we get $(a + 1) \times (b + 1) \times (c + 1)$ pieces of the original cube. These pieces may or may not be cubes and that essentially depends on whether the cuts are equidistant or not and also on whether a, b and c are equal. The following cases occur:

  Case 1: All the pieces are cubes when a = b = c and all the cuts are equidistant.

Case 2: All the pieces are not cubes when a = b = c and all the cuts are not equidistant.

Case 3: All the pieces are not cubes when a ≠ b ≠ c.

Learning Point: To maximise the total number of pieces, with the minimum number of cuts, we need to minimise the difference between the number of cuts in any two axes.

EXAMPLE

What is the least possible number of cuts required to cut a cube into 70 identical pieces?

(a) 35 (b) 15
(c) 17 (d) 11

In order to think about this, you need to see the various ways in which the given number 70, can be broken into three factors. You can visualise 70 as:1 × 1 × 70; 1 × 2 × 35; 1 × 7 × 10; 1 × 5 × 14; 2 × 5 × 7. Since, the three component factors are closest in 2 × 5 × 7, we will choose that set of factors for 70, in order to get the minimum number of cuts.

Then, the following thought process would apply:

$70 = 2 \times 5 \times 7 = (a + 1) \times (b + 1) \times (c + 1) = (1 + 1) \times (4 + 1) \times (6 + 1)$.

Least possible number of cuts = 1 + 4 + 6 = 11. Hence, option (d) is correct.

LEARNING POINT

To minimise the total number of pieces, we need to maximise the difference between the number of cuts in any two axis.

EXAMPLE

What is the maximum possible number of cuts required to cut a cube into 70 identical pieces?

(a) 55 (b) 45
(c) 77 (d) 69

In order to think about this, you need to see the various ways in which the given number 70, can be broken into three factors. You can visualise 70 as: 1 × 1 × 70; 1 × 2 × 35; 1 × 7 × 10; 1 × 5 × 14; 2 × 5 × 7. Since, the three component factors are farthest in 1 × 1 × 70, we will choose that set of factors for 70, in order to get the maximum number of cuts.

Then, the following thought process would apply:

$70 = 1 \times 1 \times 70 = (a + 1) \times (b + 1) \times (c + 1) = (0 + 1) \times (0 + 1) \times (69 + 1)$.

Maximum possible number of cuts = 0 + 0 + 69 = 69. Hence, option (d) is correct.

III. Painting and Cutting a Cube:

When we paint a cube and then make cuts along the axes of the cube, you are often asked to visualise the number of smaller cubes that have three faces painted, two faces painted, single face painted or no faces painted. The best way to work out such problems is through the formulae listed below. However, while I would want you to actually remember these formulae, I would also want you to visualise these outcomes for an actual problem.

A: Cutting equidistant along each axis with an equal number of cuts.

Let us say, the cube is painted black. If we make n – cuts in each axis then:

Number of cubes with 3 faces painted black = 8 (all corners painted cubes).

Number of cubes with 2 faces painted black = $(n - 1) \times 12$.

Number of cubes with 1 face painted black = $(n - 1)^2 \times 6$.

Number of cubes with no face painted black = $(n - 1)^3$.

Thus, if we make 2 equidistant cuts on each edge of the cube, we would get 27 smaller cubes. Out of these 27 smaller cubes, 8 cubes would be painted black on three sides; $(n - 1) \times 12 = (2 - 1) \times 12 = 12$ cubes painted black on 2 sides; $(n - 1)^2 \times 6 = (2 - 1)^2 \times 6 = 6$ cubes painted on 1 side and $(n - 1)^3 = (2 - 1)^3 = 1$ cube having no sides painted.

For n = 3, there would be 64 smaller cubes formed. In such a case, the number of cubes having:

3 sides painted = 8;
2 sides painted = $(n - 1) \times 12 = (3 - 1) \times 12 = 24$;
1 side painted = $(n - 1)^2 \times 6 = 24$ and
No side painted = $(n - 1)^3 = (3 - 1)^3 = 8$.

Please try to do your visualisation too for these cases, so that you get a better understanding and grip on the problem.

B: Cutting equidistant along each axis with an unequal number of cuts.

If the number of cuts are not equal, let's say we have made 'a' cuts on the X – axis, 'b' cuts on the Y – axis, and 'c' cuts on the Z – axis, then, first of all, the smaller pieces formed would not be cubes – they will be cuboids. In such a case, the questions can be framed asking us to determine the number of cuboids that have 3 faces painted, 2 faces painted, 1 faces painted and no face painted.

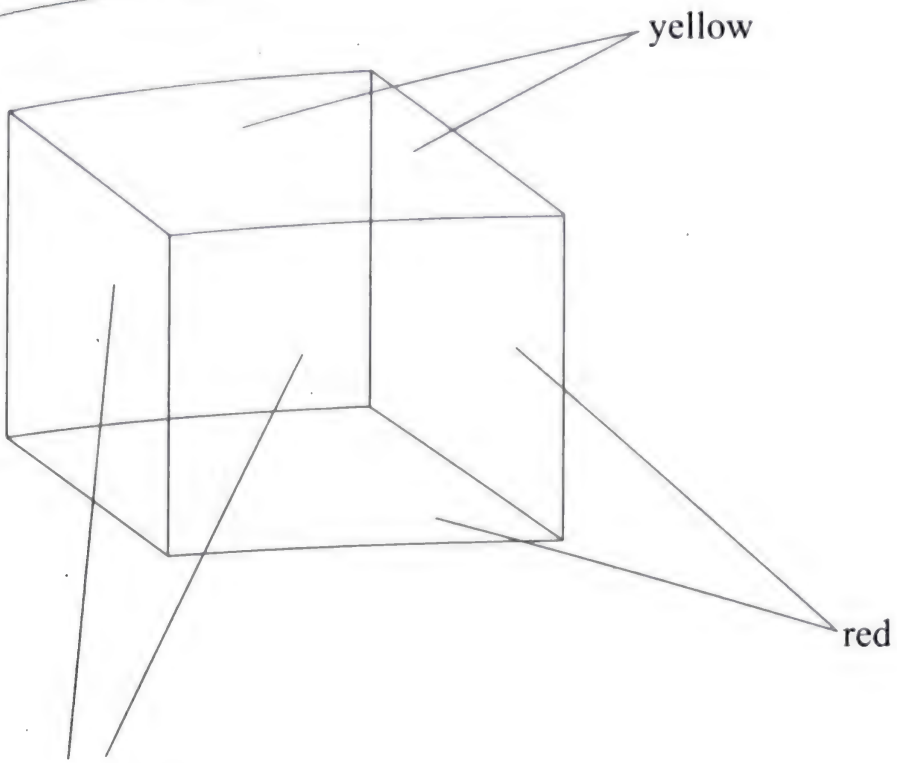
Number of cuboids with 3 faces painted black = All the corner cubes, which is equal to 8 cubes.

Number of cuboids with exactly 2 faces painted = $(a - 2) \times 4 + (b - 2) \times 4 + (c - 2) \times 4$.

Number of cuboids with exactly 1 face painted = $2 \times [(a - 2)(b - 2) + (b - 2)(c - 2) + (a - 2)(c - 2)]$.

Number of cuboids with 0 face painted black = the inner cuboid = $(a - 2) \times (b - 2) \times (c - 2)$.

One more thing you need to visualise for yourself is if the faces of the cubes are not painted with the same colour - i.e., if the faces are painted with different colours, such that each colour is used to paint 2 faces:



We make the following observations:

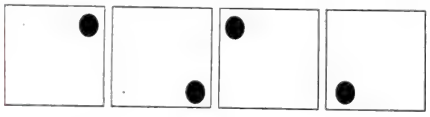
1. Number of cubes with all the three – colour faces = 2 cubes diagonally opposite to each other.
2. Number of cubes with only same colour on two faces = three edges without corner cubes = $3 \times (n - 1)$.
3. Total cubes on the edges = $4 \times (n + 1) + 8 \times (n - 1)$.
4. Number of cubes with only 2 colour faces = total cubes on edges – same colour on two faces cubes – all three colour face cubes.
5. Number of cubes with exactly one face red = cubes on the two faces of the painted red surface = $2 \times (n - 1)^2$.
6. Number of cubes with faces only red = $(n - 1) + 2 \times (n - 1)^2$.
7. Number of cubes with exactly one face painted = $6 \times (n - 1)^2$.
8. Number of cubes with exactly two surfaces painted with different colours = $9 \times (n - 1)$.

Non-Verbal Reasoning

Questions on non-verbal reasoning are based on finding the patterns in related figures. These questions are an important part of preparation for CSAT exam and the 2014 paper contained 4 questions on this pattern.

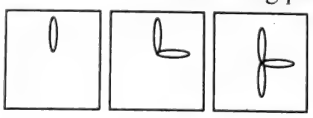
The general process of solving a non-verbal reasoning question involves looking for consistent patterns from one picture to the next. Some of the most common patterns which are used frequently are as follows:-

1. **Movement of a part of the picture in a consistent pattern:** For example, in the figure given below, the circle is moving clockwise by one place, then by two places, then three and so on.

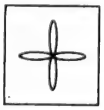


Obviously, in the series, the next figure will have the circle in the same place.(after moving four places clockwise)

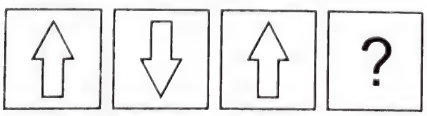
2. **Consistent pattern of change in a part of the picture:** Have a look at the following pattern.



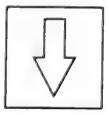
In the above pattern, it is clear that one extra petal is being added to the flower. Hence, the fourth figure will be expected to be



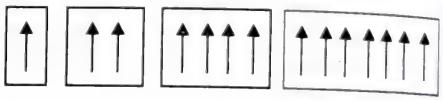
3. **Pattern of reversal of the picture:** For example, have a look at the following pattern



In the above pattern, it is clear that the arrow reverses every time. The missing figure will be

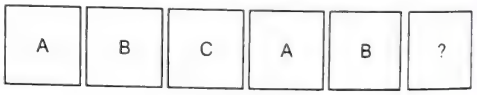


4. **Pattern of addition to a figure:** Consider the following example,

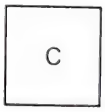


A close look at the above figure tells us that the number of arrows is increasing by +1, +2, +3, ...
Hence, the next figure will have 11 arrows pointing upwards.
[**Note:** Pattern of deletion: Would be reverse to the pattern of addition & can be visualised as deletion of certain parts of a picture, in an ordered fashion.]

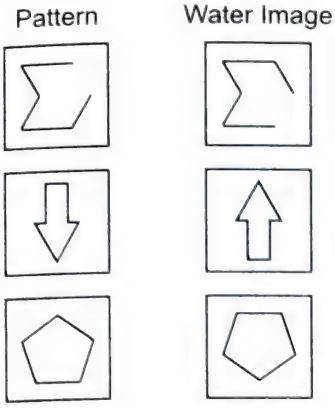
5. **Pattern of Alternation:** Consider the following example,



In the above series, there is an alternating value in the picture. There are 3 values viz. A, B, C which are alternating. Hence, the missing figure will be

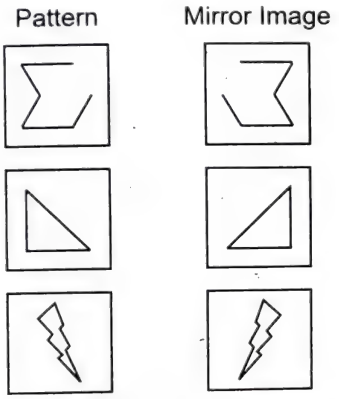


6. **Water image:** In order to visualise water image, the best approach is to try to draw the original image using wet paint and then visualising where the imprint would form if the paper is folded horizontally. Have a look at the following figures



We encourage you to try to visualise more situations for water images.

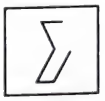
7. **Mirror Image:** The same wet paint principle for Water image can be utilised to visualise mirror images. The only difference is that the folding of the paper is vertical



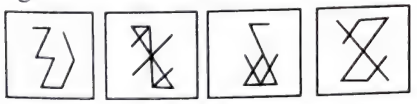
We encourage you to try to visualise more situations for water images.

All the above patterns are visible in the questions asked in the CSAT 2014 pattern as shown below.

EXAMPLE 1 Examine the following figure:



Which one of the following figures has the above figure embedded in it?

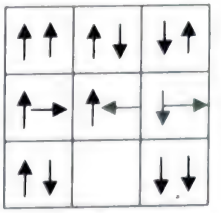


(a) (b) (c) (d)

SOLUTION

For the above question, the correct answer is option (c). In the given example, the figure is embedded as it is within the answer option. However, while solving these kinds of questions, you need to also visualise the various possibilities of the water image or the mirror image of the figure being embedded.

EXAMPLE 2 Consider the following matrix:



Which one of the following figures fits into the blank part of the above matrix?

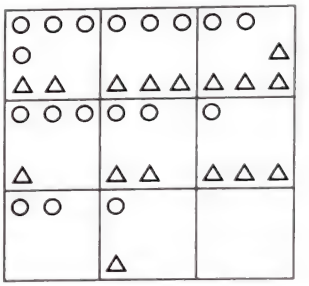
(a)  (b) 

(c)  (d) 

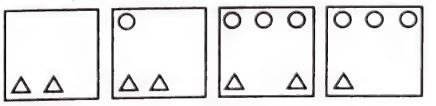
SOLUTION

If you look at the question figure row-wise, it can be visualised from the figure that in each row, going from the first figure to the second (on it's right) there is a reversal of the second arrow's direction. From the second to the third figure in each row, the consistent rule that can be made out based on the first two rows is that both the arrows are reversed. Hence, we can infer that the correct answer for the missing figure should contain two arrows facing upwards. Option (b) is correct.

EXAMPLE 3 Consider the following matrix with one empty block in the lower extreme corner:



Which of the following figures could fit in the empty block and thus complete the matrix?



(a) (b) (c) (d)

SOLUTION

In this question we can clearly see that the circles are following a pattern of deletion (-1 going from left to right) and the triangles are following a pattern of addition (+1 going from right to left). Hence, the correct answer should have no circles and 2 triangles. Option (a) is correct.

EXAMPLE 4 Consider the following figures:

Change in positions of beads in the four figures above follows a sequence. Following the same sequence, which of the figures below should appear as the fifth figure above?

(a) 

(b) 

(c) 

(d) 

SOLUTION

In this figure, there is a movement pattern being adopted for each of the four kinds of circles. For instance, the circle with it's left part shaded, starts from the top left vertex of the triangle and from one figure to the next is being moved anti-clockwise by 1 position. So it is seen at the bottom left, then bottom right and finally at the top right in three consecutive movements. Hence, in the correct answer it should be back at it's original position. Each of the other three shaded circles are following either a clockwise (+1) position change, or an anti-clockwise (-1) position change. In the fifth figure of the series, all the circles should be back at their original positions. Hence, option (b) is correct.

Introduction

The test of 'decision making and problem solving' is a unique aspect of the Civil Services (Preliminary paper II) Examination. Its objective is to go beyond the linguistic, numerical and other abilities which are usually assessed in aptitude tests and evaluate those set of traits which are integral to becoming a competent and upright civil-servant. Taking decisions is an inseparable and critical part of the work profile of a civil-servant. And often these decisions are to be exercised in environments of ethico-legal-practical dilemmas or acute physical constraints. How would a candidate respond to such situations is what this topic endeavours to evaluate.

The questions in this section can broadly be categorised into those substantially dealing with, either 'decision making' or with 'problem solving'. Although both sub-categories may sound one and the same thing, or similar to each other, there is a salient difference between them. While both involve situational questions (where a situation is given), the former requires evaluation of the ethical and moral or even legal dimensions of the situation and taking a decision according to the values/ principles entailed. On the other hand the latter requires analysing a given situation to overcome a matrix of constraints or conditions. The salient aspects of both category of questions, along with illustrative examples are discussed below.

1. DECISION MAKING

As the very name implies this section tests the candidates ability to take an appropriate decision in a given situation. The hypothetical situation described in the question may be picked up from a variety of real life situations and need not necessarily be only those encountered by the civil servants. Thus, a question may be based on a situation encountered by a common citizen in day-to-day life. But what all of these aim to evaluate is, whether the values and abilities which are displayed by the candidate in arriving at a decision, are in consonance with the principles which ought to govern the decision making process of an upright and competent civil-servent. Thus, what needs to be comprehended in each of these questions is the **underlying value/ principle** which the given situation requires you to identify and dwell upon.

1.1 Features of Questions

Nature of questions The questions will deal with different real life situations. It will describe who you are and what is the situation (viz. A District Collector dealing with flood relief; a doctor dealing with supply of medicines/ vaccines; an executive of an MNC dealing with the launch of a product etc. The question will then present a dilemma or a situation of difficulty in which a course of action has to be adopted or a decision taken. This can be, the course of action to be adopted by the Distt. Collector where merchants are charging double the usual price for relief material; or in case of scarcity of medicine, as a doctor which patient to accord priority; or as an executive of a MNC whether to choose corporate responsibility or corporate profit in case of conflict of interest between the two.

This is followed by four options each describing a different course of action to deal with the situation. You are required to indicate your choice as to the most preferred course of action, if your were in that given role.

Marking of questions The General Studies Paper II of 2011 contained eight questions from this section. Each of these questions were of 2.5 marks. Thus this section was allotted a total of 20 marks or 10% of the weight of the entire paper. The unique feature of the 'decision making' questions was that there was no negative marking for these eight questions, while the paper II had negative marking for the rest of the questions. This implies, that unlike other questions in the paper, **this set**

of questions will not necessarily have only one option which is right while other options are absolutely wrong. Rather, the four choices which are given **may actually be graded ranging from the best to the worst choice.** Thus, for exercising different options one may score marks ranging from 2.5 to 0 (viz. 2.5 marks for choice (b), 1.5 marks for choice (c) 1 more for choice (a) and 0 mark for choice (d).

1.2 Abilities Tested

The questions in this section are of such a nature that they entail ethical and moral dimensions of decision making. As mentioned earlier, each of these questions present a decision making dilemma or difficulty. The key to selecting the right option lies in understanding which principle or value is entailed in the situation given. This forms the nucleus of the question around which the described circumstances and possible course of actions revolve. Some of these values/ principles are;

- Upholding truth and justice
- Upholding law and observation of rules
- Honesty and integrity
- Empathy for people
- Sense of responsibility and sincerity
- Initiative in work and devotion to duty
- Valuing merit despite pressure
- Valuing transparency and probity in public life
- Valuing ethics and morality in public life
- Not succumbing to extraneous pressures and influences in course of discharge of duties.

The list of course is not exhaustive. What is pertinent is to first identify which of these or other values or principles are entailed in the situation. Then the most important thing is to ascertain which of the given options best upholds the aforesaid principles or the values. At times the practicability or feasibility of the option is also to be seen and a fine balance has to be struck between the principles and its applicability in the given situation. Thus, it is not merely the identification of the values but also the sense of balance of judgement which is put to test.

1.3 Illustration

You have rented out your flat to a tenant who has not paid you rent for the past six months and now even refuses to vacate your flat. He has offered to purchase that flat from you but only at 75% of the present day market value. He flouts his contacts and has threatened you of dire consequences if you lodge a police complaint. What do you consider the most appropriate course of action?

(a) Lodge a police complaint for unauthorised occupation of the flat and for threatening you.

(b) Negotiate with the tenant for a higher price and sell it, as taking legal recourse is time consuming and arduous.

(c) Approach local goons and pay them to throw him out of the flat.

(d) Look out to sell the flat to a third person at slightly less than the market price (but higher than what the tenant is offering) and get away from the mess.

Ans. 2.5 marks if choice is (a)

1.5 marks if choice is (d)

1 mark if choice is (b)

0 mark if choice is (c)

Rationale/ Reasoning

- Taking legal recourse is the best course of action despite being arduous and time consuming. Such a choice **indicates readiness to fight for justice despite odds**.
- Selling a flat to a third party, though better than the remaining two options, it still indicates lack of desire to fight for justice and also is an escapist approach. Besides, a ready buyer may not be available for an unauthorisedly occupied flat. This may cause loss of time and force you to revert to action at choice (a), hence it is not the best course of action.
- Selling the flat to the tenant is an escapist approach and helps in perpetuating an injustice.
- Approaching local goons to throw the tenant out is an illegal act and absolutely incorrect. It is akin to employing one wrong to correct another wrong.

Principles/Values Tested

Belief in the rule of law and desire to fight against injustice.

1.4 How to Answer?

- Read the question properly and understand what is your role, what are the constraints/dilemmas which have been expressed in the question and what are the given choices.
- **Identify which of the values and principles does the question entail**. Also identify which of the options best **upholds** the principle. In the example cited above, it is the belief in the 'rule of law' which is at the core of the issue and option 'a' best reflects this belief.

- Examine the practical applicability of the choice you think best upholds the value/ principle. But be cautious, not to discard this choice unless it is absolutely impracticable or its application will defeat the very purpose of the action/work which is required to be performed.
- The best option may often have some difficulty associated in its implementation. This may be incorporated in the question or the options in form of, pressure from the boss, need for sacrifice of personal interest, long and arduous course of action to fight for justice etc. **These difficulties should not be misconstrued** to be impracticability. In the example cited such difficulty is the long and arduous procedure for seeking justice.
- To identify/ narrow down the choices, **apply the technique of selection by elimination**'. In most of the questions just after a preliminary reading of the given options you would be able to identify at least one or even two options which can be rejected outrightly, as they do not confirm to the principles involved. You have to use your balance of judgement to select the best option in the remaining two choices.
- In case you continue to be besieged by dilemma as to what is the best choice, then test the options on the touchstone of ideals and propriety. Ask yourself, what trait or values is being reflected if a particular option is chosen? Would this behaviour/ trait befit or is desirable for a civil servant? Please remember that an **examiner would always like to select a candidate who shows belief in ideals and principles rather than one who is cynical, fatalistic or ready to compromise on ethics, at the very outset of his career**. Though, as a common citizen one may make some compromises but for a civil servant the bar is raised much higher.
- As the decision making questions **do not entail negative marking** and are likely to have a graded marking for various choices, it advised that you **attempt all the questions** in this section.

2. PROBLEM SOLVING

The questions in this section are likely to test a candidate's ability to understand and assimilate facts and to take decisions/figure out an optimal solution to problems, in which a set of constraints have been provided. Thus, what the term 'problem' refers to here, is essentially a matrix of conditions or constraints which are to be overcome and not the typical mathematical/ numerical problems which we usually encounter. The question structure and other salient features, illustrative example and a variety of unsolved questions are being provided to help you familiarise with this topic.

2.1 Question Structure

The structure of the questions on 'problems solving' can be divided into the following parts;

(a) Situational facts
(b) Matrix of conditions/constraints
(c) Question Statement
(d) Decision options

The questions on this topic are usually built upon a background of facts which are similar to a real life situation. These are called 'situational facts'. For example in a problem pertaining to 'inventory management', the level of inventory (as described in the question) can be said to be the **'situational fact'**. The question will then specify the conditions to be followed for inventory management or minimum level to be maintained. These are referred as the **set of constraints**.

The next part is the **'question statement'** which indicates what is the candidate required to do, i.e. what is the decision/ the optimal solution he has to arrive at. The last part of the question is the **'decision options'**, which are in the form of the four answer choices. The candidate is required to consider the 'matrix of constraints', the 'situational facts' and take a decision as to which is the best option within the given constraints. To illustrate the above a solved example is being given below. This is followed by tips and techniques helpful in solving such problems.

2.2 Illustration

Mr. A is the head of HR Division of a company which has employees in three different categories, group A, group B and group C. The number of employees in each category is indicated in the table below. Certain employees in each category have submitted their resignation and are quitting the company from next month (see table). However, for the functioning of the company, maintaining the same number of employees is essential. Therefore the HR Manager has to fill those vacancies either through fresh recruitment in the three categories or vide promotion from group B to group A and group C to group B.

In direct recruitment as well as in promotions, certain expenses are incurred. These the HR Manager has to consider and has to come up with the most economical solution to fill in the vacancies. The cost of recruitment in category A, B, C is ₹ 10,000/-, ₹ 5000/- and ₹ 2000/- respectively. The cost of promotion of a group C employee to group B is ₹ 1000/- and that of group B employee to group A is ₹ 3000/- per person (expenses are incurred on promotion and training).

If the recruitment/promotion exercise involves a total cost of ₹ 20,000/- or less then the HR Manager has the authority to do it on his own. If the exercise costs ₹ 20,000/- to ₹ 40,000/-, then he has to take the approval of the Finance Department and if it costs ₹ 40,000/- or more, then approval of the Finance Department as well as the company's President is required. In no case more than 50% of the employee strength of a group should be promoted at one time.

S.No	Department	Group A	Group B	Group C
1.	No. of employees working in the company	5	15	30
2.	No. of employees quitting the company	2	5	7
3.	Cost of direct recruitment of an employee	₹ 10000/-	₹ 5000/-	₹ 2000/-
4.	Cost of promotion of an employee from that category to a higher category	------	₹ 3000/-	₹ 1000/-

Attempt the Following Questions

Q.1 In the most economical model of this exercise how many employees will have to be recruited for group C?

(a) 7 employees

(b) 12 employees

(c) 14 employees

(d) Cannot be decided on the basis of the data given.

Q.2 What would be the best combination of promotion and recruitment for filling up the vacancy in group A?

(a) Recruit two persons directly in group A.

(b) Recruit one person in group A directly and promote one person from group B.

(c) Promote both the persons from group B to group A.

(d) Vacancy cannot be filled because of financial constraints.

Q.3 What is the most economical total cost of filling all of the five vacancies in group B?

(a) ₹ 10,000/-

(b) ₹ 15,000/-

(c) ₹ 20,000/-

(d) None of the above

Q.4 For filling up all the vacancies, the HR Manager will;

(a) not require anybody's permission and fill the vacancies with his own authority.

(b) will require only approval of the Finance Division.

(c) will require the approval of the Finance Division as well as the President of the company.

(d) The data is insufficient to come to a conclusion.

SOLUTIONS

- The basic constraint in the above matrix is the cost of exercise of recruitment/ promotion. This has to be done in the most economical manner and approval of the appropriate level/ authority is to be taken.
- There are seven vacancies in group C. This cannot be filled up by promotion as there is no lower category of employees. Therefore, it has to be filled up by direct recruitment in group C. The cost to be incurred is $7 \times 2000 =$ ₹ 14,000/-.
- The vacancy in group B can be filled either by recruitment, the cost of which is ₹ 5000/- per person. Employees can also be promoted from group C to group B but this will create a vacancy in group C, which will have to be filled by direct recruitment in group C. Therefore cost of filling one vacancy in group B through promotion is; ₹ 1000/- (promotion cost of a group c employee) + ₹ 2000/-(direct recruitment cost in group C) = ₹ 3000/-.

  The latter works out to be cheaper than direct recruitment. Therefore total cost of filling up five vacancies in group B will be ₹ 3000 × 5 = 15,000/-.

  A group A vacancy can be filled by;

  - Direct recruitment in Group A, the cost of which is ₹ 10,000/- per employee, or
  - By promotion from group B. The cost is ₹ 3000/-. But on promotion from group B to group A, a vacancy is left in group B. The minimum cost of filling this vacancy as worked out above is ₹ 3000/-. Therefore the most economical method of filling a vacancy in group A, would be by promotion from group B. The total cost of which will be ₹ 6000/- (₹ 3000/- cost of promotion from group B) + ₹ 3000/- (cost of filling the group B vacancy).

Answer 1. Number of group C employees required to be recruited will be 14.

Seven recruitments to fill the direct vacancy in group C.

Five to fill the vacancy in group B, as five group C employees have to be promoted.

Two to fill the vacancy in group, A as two will have to be promoted from group B, which will create two vacancies to be filled from Group C.

The promotions from group C are seven which is less than 50% of group C strength, hence, it is permissible.

Answer 2. Option (c); As shown in the working above.

Answer 3. Option (b); ₹ 15000 as shown in the working above.

Answer 4. Option (c); total cost of filling up vacancy is ₹ 41,000/-

2.3 Tips and Techniques of Solving Questions

- Read the question very **carefully**. Do not cursorily peruse the question but at each stage of the question register what it is implying before proceeding further.
- Identify and **understand what are the constraints, situational facts and decisions to be taken**. In the aforesaid example, the HR structure of the company, the number of employees leaving the company, the cost of recruitment at each level, represent the 'situational facts'. The requirement of maintaining a minimum number of employees in each category, the limit to the financial costs which the HR manager/ Finance Department/ the President can incur on recruitment, represent the matrix of constraints.
- Always **segregate the constraints/conditions** to be fulfilled from the situational facts. If the **constraints are many in number then underline each of them to make identification easy.**
- Always keep the constraints in mind while working out the solution.
- Check whether your solution fulfills each of the given constraints or not. Tick those which are fulfilled. This makes decision making simpler.
- Though most of the facts stated in the question will be used in solving the problem, some facts may not actually be utilised. These may have only been given to build/ complete the problem. However always ascertain/ check whether the fact is useful or not even though it may appear to be superfluous.

EXERCISE ON DECISION MAKING

(1) *Your immediate supervisory authority had taken some decisions regarding setting up of a project which has run into heavy revenue losses. The higher authorities have set up a three member committee to enquire into the matter which has called you for your deposition. Your boss wants you to justify the stand taken by him. He is likely to remain your boss for atleast next one year. You would,*

(a) justify the decision of your supervisory authority and bail him out.

(b) outrightly criticise his decisions as in the past he has been rude and harsh with you.

(c) give a fair account of what has happened which is likely to result in his implication, but is surely to adversely affect your personal equations with him.

(d) give an ambivalent reply showing your lack of specific knowledge of the subject and thus save you the embarrassment of implicating your boss and also save you from being further dragged as a witness against your boss.

(2) *You are a Deputy Secretary in a Ministry of the Government of India where you are dealing with appointments of 'Technical Assistants'. On the basis of a written examination, 10 candidates have been shortlisted for two posts. Based on their educational and previous work experience you have to make your recommendation in order of preference. A senior official in the Ministry who is known to you, asks you to recommend the name of candidate Y amongst the first two choices. What will you do?*

(a) Bring the matter to the knowledge of your immediate superior officer and do as he verbally directs.

(b) Comply to the request of the senior officer who approached you and recommend the name of the candidate in the top two positions despite knowing that he is not among the best two. This way you earn the goodwill of the officer who approached you.

GENERAL MENTAL ABILITY

General Mental Ability has a constant presence in the GS paper of the preliminary examination over the past decade. Before the introduction of the Civil Services Aptitude Test in 2011, the portion for the GS paper in the prelims exam always had general mental ability as a constant presence in it.

In this avatar, the GMA consisted of almost the entire portion of basic numeracy, data interpretation, logical and analytical reasoning, etc., which are in combination a significant portion of the Civil Services Prelims Paper II exam syllabus—(comprehension, decision making and English comprehension and interpersonal and communication skills being the only sections recently introduced).

In this book, we have created separate parts for basic numeracy and data interpretation, logical and analytical reasoning and general mental ability. Hence, in the scope of this part of the book, we would not be covering sections like 1) basic numeracy, 2) data interpretation, 3) data sufficiency and 4) logical and analytical reasoning. However, other types of general mental ability questions which cannot be categorised under any of the four categories above still will form a part of our scope in this part of the book.

Before we start to study the individual parts of the syllabi/question types that can come exclusively under the GMA section (after removing the overlapping areas), let us first go through the typical questions that have been appearing in the GS paper over the past five years. In order to comprehend what general mental ability means, this might be the best approach.

Please have a close look at each of these questions – because they represent an invaluable resource for deciphering what kinds of skills they are likely to test in the aptitude exam of the Civil Services prelims in its new avatar of the first few years. Hence, they also represent an invaluable resource to direct you towards the kinds of skills you need to develop as you prepare yourself for this all important test.

CHAPTER 1

Syllogisms

SYLLOGISMS—THEORY

Syllogisms can be defined as a deductive scheme under which a formal argument is made. It consists of a major and a minor premise, leading up to a conclusion.
For example, let us look at the following example:
Major Premise: Every crime is deplorable;
Minor Premise: Eve teasing is a crime;
Conclusion: Eve teasing is deplorable.

Questions based on Syllogisms always first state the premises and then ask you to derive the conclusion on the basis of the relationship existing between the different elements of the premises.

The best method for solving syllogisms involves the use of Venn Diagrams. We will first look at the main types of premises and then look in detail at the process to be used to solve questions of syllogisms.

PREMISES—VARIOUS PATTERNS

A premise is defined as a proposition antecedently supposed or proved as a basis of argument or inference. In other words: something assumed or taken for granted.

Type 1: All A's are B's (Or No A is not B):

There are two possible Venn diagrams for this situation. These are:

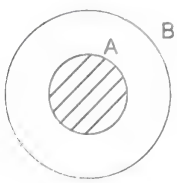


Fig. A

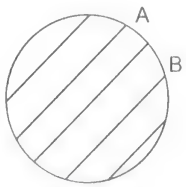


Fig. B

The following reactions to the premise *All A's are B's* are valid:

(a) *Some B's are A's.* This is a definite conclusion. This is true in both Figures A and B.

(b) *Some B's are not A's.* This is a probable conclusion and will occur only if the conclusion *All B's are A's* is not true.

Hence, it can be stated that if All A's are B's, then either All B's are A's (No B is Not A) or Some B's are not A's. As can be seen in the figures above, either *Some B's are not A's* (Figure A) or *All B's are A's* (as seen in Figure B).

Type 2: Some A's are B's

This premise is represented by the following figure:

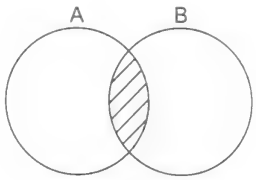


Fig. A

The valid reaction to this premise is:
Some B's are A's

This is a definite conclusion (as can be seen clearly in figure A above.)

Although the above figure also supports the conclusion —some B's are not A's, this cannot be taken as a definite conclusion. This is because, when we say that *Some A's are B's,* it does not mean that there have to be some B's that are not A's.

Type 3: No A is B

This premise is represented by the following figure:

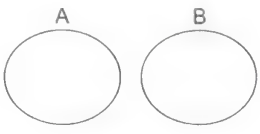


Fig. A

The conclusion *No B is A* is a valid conclusion.

Type 4: Some A's are not B (Or All A is not B)

There could be three possible Venn Diagrams in this case, as shown below:

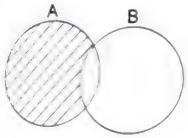


Fig. A

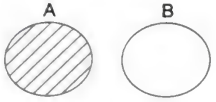


Fig. B

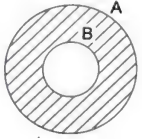


Fig. C

STANDARD PROCESS FOR SOLVING SYLLOGISM QUESTIONS

The most logical process of solving Syllogism questions is through Venn diagrams. The following examples will make the process clear:

CHAPTER 2

Logical Deductions

As the name itself suggests, logical deduction questions require logical thinking. Every question starts off with a premise which might be of any one of the following types:

LOGICAL DEDUCTIONS PREMISES—VARIOUS PATTERNS

Type 1: If A Happens B Happens

This means that A leads to B, but does not mean the reverse, i.e., if B has happened, A must have happened. In such situations, A is a sufficient condition for B, but is not a necessary condition.

Let's look at the following example:

If I go to a movie, I enjoy myself.

This would mean, that if I have gone to the movie, I will definitely enjoy myself. However, it does not mean that if I have enjoyed myself, I must have gone to the movie—there are so many ways of enjoying yourself.

Another example of this type would be:

If Amitabh acts in a movie, he will earn money.

This does not mean that if he has earned money, he must have acted in the movie.

Type 2: Only If A Happens B Happens

In this case, A is a necessary and sufficient condition for the occurrence of B. In this case, there is reversibility of the logic, i.e., if B has happened, A must have happened.

Only if Saurav plays the match, will he get a hundred.

If Saurav has got a hundred, he must have played the match.

Type 3: If A Happens B does not Happen

In this case, if A has happened, B does not happen. The opposite is also true, i.e., if B has happened, A must not have happened.

If Martina plays well, Sania will not win.

Type 4: If A does not Happen B will Happen

Again in this case, the reverse might not be true.

If Anand does not come, Kasparov will. This does not mean that if Kasparov comes, Anand will not come.

Type 5: Either A or B will Happen

One of the two has to happen. At the same time, the two events are exclusive of one another. If one happens, the other will not happen.

Either he becomes a TV star or he becomes a movie star.

Let us look at a few solved examples:

1. Shahrukh either acts as a villain, or he acts as a hero.
   A. Shahrukh acts like a hero.
   B. Shahrukh does not act like a villain.
   C. Shahrukh acts like a villain.
   D. Shahrukh does not act like a hero.
   (a) CD only (b) BA only
   (c) CD & BA (d) None of these

This question is based on an EITHER-OR Premise. Thus, we can see that CD is correct. Since, if he acts like a villain—(Statement C) he will not act like a hero (Statement D).

Similarly, BA can also be seen to be true. Hence option (c) is correct.

2. Whenever Martin goes to a movie, he has nightmares.

A. Martin did not have nightmares.
B. Martin went to a movie.
C. Martin had nightmares.
D. Martin did not go to the movie.

(a) AD (b) DC
(c) CB (d) DA

When A happens, B happens. However, if A does not happen, it is not necessary that B will not happen. Also, if B has happened, it is not necessary that A must have happened. However, if B has not happened, it is necessary that A must not have happened.

Thus, AD is the only correct sequence.

3. If I talk to my girl friends, then I do not need to take a pill for heartache.

A. I talked to my girl friends.
B. I did not need to take a pill for heartache.
C. I needed to take a pill for heartache.
D. I did not talk to my girl friends.

(a) AB only (b) DC only
(c) CD only (d) AB and CD

This question falls under the category of if A happens, then B does not happen. However, if B has not happened, it does not mean that A has happened.

AB and CD are both logically correct.

CHAPTER 3

Statements and Conclusions

Conclusions are inferences that can be drawn on the basis of the information contained in a paragraph/statement. A conclusion in other words is also an inference that follows the given information. Drawing conclusions is something we all do regularly. The main difference between a conclusion which we saw in the last chapter and an assumption is that while an assumption precedes the information a conclusion follows it.

The following solved examples will help you better understand conclusions:

Directions: In each of the following questions, a statement is given followed by two conclusions. Give answer

(a) If only conclusion I follows.
(b) If only conclusion II follows.
(c) If both conclusions I and II follow.
(d) If neither I nor II follows.

EXAMPLE 1

Statement

Ahmedabad has a lower population and hence is a safer place to live.

Conclusion

I. A city which has less population is a better place to live.
II. Ahmedabad has lower number of crimes than Bombay.

SOLUTION

Both the conclusions do not follow. The first conclusion statement is in fact an assumption on which the argument statement is based. It is not a conclusion which can be derived from the statement. Similarly, we have no information about crime rates in Bombay in the statement and hence conclusion II also does not follow. Hence, option (d) is correct.

EXAMPLE 2

Statement

All students of my class have a bright chance in their examination.

Conclusion

I. I teach them the whole syllabus thoroughly.
II. All the students are intelligent.

SOLUTION

A teacher can claim that his students have a bright chance in the examination only if he teaches the syllabus thoroughly. However this is an assumption that must be valid for the statement to be made (and is not a conclusion). Hence conclusion I is valid. Conclusion II is also invalid because the 'students' who are being referred to are not defined clearly. Hence, option (d) is correct.

EXAMPLE 3

Statement

Anyone who manages an engineering organisation like Tata Motors can only be successful if he has the knowledge of the professional work that is carried on in the organisation.

Conclusion

I. If you want to run any company, it is essential that you should know the professional work associated with it.
II. In order to manage an engineering organisation like Tata Motors, it is essential to have an engineering education background.

SOLUTION

The statement particularly talks about engineering organisations like Tata Motors. The first conclusion is generalising the statement and is hence not valid as a conclusion. The second conclusion follows. Hence option (b) is correct.

EXAMPLE 4

Statement

According to Baba Ramdev "Values, integrity and peace of mind are essential for a beautiful life."

Conclusion

I. Baba Ramdev believes that each of values, integrity and peace of mind are essential for a beautiful life.
II. Baba Ramdev believes that materialistic persons can not have a beautiful life.

SOLUTION

Conclusion I follows since that is what is meant by the statement. Materialism is generally anti values and integrity and hence we can draw the second conclusion. Hence option (c) is correct.

EXAMPLE 5

Statement

India won the second test match against Pakistan by one innings and two hundred and three runs. The greatest contribution was from the bowlers.

Conclusion

I. Most of the Indian players are bowlers.
II. It was India's biggest win against Pakistan.

SOLUTION

Both conclusions are invalid. Hence, option (d) is correct.

CHAPTER 4

Statements and Assumptions

Whenever we say something (in any conversation) there are certain assumptions which must be true for the statement to make sense. In other words an assumption is something that is 'taken for granted' in the context of a statement.

Most questions on Statement and Assumptions give you a statement and follow it up with two further statements which have to be tested for whether they are implied in the original statement or not. In order to identify whether an assumption is implied in a statement the best way to do so is to look at the original statement in the question by negating the assumption which is being tested. In case the assumption being tested is implied, negating it will have the effect of weakening the argument of the original statement.

Directions: In each question below is given a statement followed by two assumptions numbered I and II. Consider the statement and decide which of the given assumption is implicit.

Given answer

(a) If only assumption I is implicit.
(b) If only assumption II is implicit.
(c) If both I and II are implicit.
(d) If neither I nor II is implicit.

EXAMPLE 1

Statement

Strong programming skills cannot be the only criteria for a successful career in the field of Information Technology.

Assumption

I. Bright careers in Information Technology are not linked only with strong programming skills.
II. If a person has excellent communication skills as well as strong programming skills, only then would he be successful.

SOLUTION

Clearly, there should be some other skills such as interpersonal skills, communication skills, negotiation skill, etc., for a successful career. Hence, assumption I is implicit. Assumption II is not implicit because it talks only about communication and technical skills. Hence option (a) is correct.

Note: Try the negation test in this question. If Assumption 1 is not true, i.e. "Bright careers in Information Technology are not linked only with strong programming skills" is untrue, then the argument contained in the statement "Strong programming skills cannot be the only criteria for a successful career in the field of Information Technology" loses its strength. Hence, this assumption is implicit in the statement.

EXAMPLE 2

Statement

The maximum IAS exam selection will be from our institute—A director of a coaching institution.

Assumption

I. Most of the students who study in this institute are genius.
II. Teachers of the coaching institution are preparing their students thoroughly.

SOLUTION

Assumption I is irrelevant because there might be some other genius students who are studying in other coaching institutions. Assumption II is implicit.

A coaching institution can only claim such statements if it is preparing its students thoroughly. Hence, option (b) is correct.

EXAMPLE 3

Statement

Over a cricketing career most players would do well to remember that there is only one thing that overcomes hard luck—hard work.

Assumption

I. If a cricketer is hardworking, he/she can easily tackle the toughest phases of his cricketing career.
II. A cricketer writes his/her own destiny by hard work.

SOLUTION

Both the statements are implicit. All cricket players face difficult times in their careers, but only those succeed, who confront these phases bravely and work hard to overcome them. In this way, hard luck becomes insignificant for a cricketer. Hence, option (c) is correct.

EXAMPLE 4

Statement

The case of Marion Jones clearly indicates that if taking drugs play a vital role in the performance of athletes, there is absolutely no use of spending time to watch the athletics in the Olympics.

Assumption

I. Cricketers, hockey players and footballers are not involved in doping. So we can spend time to watch such games.

II. Asian athletes do not win medals because they refrain from taking drugs.

SOLUTION

Both the assumptions are irrelevant. In the statement nothing is said about other games and as such negating that assumption has no effect on the strength of the argument in the statement. Besides, the performance of Asian athletes is irrelevant to the argument. Hence, option (d) is correct.

EXAMPLE 5

Statement

A Reliance Fresh Retail store was attacked by vegetable vendors in Ranchi.

Assumption

I. Reliance Fresh has affected the livelihood of local vegetable vendors.

II. Reliance Fresh stores are built in thickly populated areas.

SOLUTION

Assumption I is implicit because somewhere down the line, Reliance Fresh is affecting the local vendors. That is why they have been violent. Assumption II is irrelevant because nothing is mentioned regarding the location of the stores. Hence, option (a) is correct.

CHAPTER

5

Assertion and Reason

An Assertion is an affirmation, a declaration or a strong statement. Normally assertions have reasons. In this question type, an assertion is first given followed by a reason. You are supposed to first judge whether the assertion (A) is true and then decide on whether the reason (R) is true and in case both A and R are true then we need to see whether the reason is the correct explanation of the Assertion.

The following solved examples will help you get a clearer understanding of this question type.

Directions: For the Assertions (A) and Reasons (R), choose the correct alternative from the following.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.
(e) If both A and R are false.

1. **Assertion** Crude oil is abundantly found in nature.
   Reason It is the main raw material for all automobiles.
   Solution Both Assertion and Reason are true but the Reason does not explain the Assertion. Hence, (b) is the answer.
2. **Assertion** Anil Kumble lost his place in the Indian ODI squad sometime back.
   Reason In spite of being India's best spinner he was considered to be too old for ODIs.
   Solution Both Assertion and Reason are right and the reason correctly explains the assertion. Hence (a) is the right answer.
3. **Assertion** Over the past few decades, there has been drastic global warming.
   Reason The push for economic development has increased to dangerous proportions the emission of greenhouse gases into the atmosphere.
   Solution Green house gases are the root cause of global warming. Hence both A and R are true and R explains A. Hence, option (a) is correct.
4. **Assertion** Kerala is the largest state in India.
   Reason Some of the leading car manufacturing companies of the world are Japanese.
   Solution The Assertion is false because Kerala is not the largest state in India. R is true but it is not related to Assertion. Hence, option (d) is correct.
5. **Assertion** The filament inside a bulb is made of copper.
   Reason Usually there is vacuum in the bulb.
   Solution A is false because a bulb's filament is made of Tungsten and not copper. R is also false because inert gases or nitrogen are filled inside the bulb and there is no vacuum in the bulb. Hence, option (e) is correct.

EXERCISE

Directions: For the Assertion (A) and Reason (R) below, choose the correct alternative from the following.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

1. **Assertion** In medical parlance with respect to blood groups, a person with a blood group of 'O' is called Universal Donor.

CHAPTER 6

Statement—Courses of Action

Most situations that we face in life throw up the possibility of creating an improvement by following certain courses of action. In fact, deciding on a course of action in a given situation is a primary responsibility in any managerial job.

This question type tests you on your ability to decide on how a situation can be tackled/improved/followed up. Look through the following solved examples to get this question type clear.

Directions: In each of the following questions a statement is given followed by two courses of action. A course of action is taken for improvement, follow up, etc. Read the statement carefully and give your answer as
(a) If only course of action I follows.
(b) If only course of action II follows.
(c) If both I and II follow.
(d) If neither course of action I nor II follows.

EXAMPLE 1

Statement

The presence of mafiosi in the education system of UP has increased drastically.

Course of Action

I. There should be a special taskforce constituted to clean the system of its ills.
II. The U.P. government should resign immediately.

SOLUTION

Course of action I follows because it is necessary to tackle such antisocial elements.

Course of action II does not follow because the resignation of the government is unlikely to improve the situation. Hence, option (a) is correct.

EXAMPLE 2

Statement

In a recent survey by the National Health mission it was showed that fruits and vegetables which contain certain vitamins have a higher effect on human health than capsules marketed by private companies that contain the same vitamins.

Course of Action

I. Such vitamin capsules marketed by private companies should be banned with immediate effect.
II. People should prefer fruits and vegetables to capsules marketed by private companies.

SOLUTION

Course of action I does not follow because we are only making a comparison between the effects of food and vegetables against the effect of capsules. Capsules have just been found to be less effective and it does not mean that they are harmful.

Course of action II follows because fruits and vegetables are preferable to capsules for their positive effect on health. Hence, option (b) is correct.

EXAMPLE 3

Statement

It has been reported that water level is declining rapidly in India.

Course of Action

I. There should be a public campaign to educate people about the need to reduce and eliminate the wastage of water.
II. New technologies should be researched and utilised to store rain water.

SOLUTION

Both the courses of action are logical steps that could be taken to address the problem. Hence, option (c) is correct.

EXAMPLE 4

Statement

Reliance Telecommunications is playing dirty tricks with its competitor Tata Indicom.

Course of Action

I. Tata Indicom should also do the same.
II. Tata Indicom should decrease the tariff rate of phone calls.

SOLUTION

Both the courses of action do not follow. Tata Indicom should not start playing dirty tricks but should approach the government and its regulatory bodies to help. The IInd course of action is also not valid because it will do nothing to stop the dirty tricks. Hence option (d) is correct.

EXAMPLE 5

Statement

There is a proposal for the Maharashtra government to clear the slum areas in Mumbai for beautification and economic development.

Course of Action

I. The Maharashtra Government should compensate the affected persons with reasonable amount.
II. The Maharashtra Government should stop beautification and economic development work immediately.

SOLUTION

Only course of action I follows. Government should take care of affected people. It should pay reasonable amount to affected persons. Second course of action does not follow because it is illogical. Hence, option (a) is correct.

CHAPTER 7

Mathematical Operations

Questions on Mathematical operations will first start with either defining a new meaning to existing symbols or define a new symbol and its mathematical meaning.

You are then required to evaluate the value of an expression by assuming the meaning of the symbols as defined in the question. The following questions will make this question type clear to you.

Illustration 1: If '+' means '–', '–' means '×', '×' means '÷', and '÷' means '+' then which of the following will be the value of the expression?

$32 + 8 \times 2 - 3 \div 4$

(a) 12 (b) 20
(c) 0 (d) 24

Answer and Explanation: Putting the changed signs, the expression can be rewritten as

$32 - 8 \div 2 \times 3 + 4$

Using BODMAS rule, we have

$= 32 - 4 \times 3 + 4$

$= 32 - 12 + 4 = 24$

Hence, the correct option is (d).

Illustration 2: If A implies '+', B implies '–', C implies '×', D implies '÷', then calculate the value of the expression $2A3B4C5D1$.

(a) 15 (b) 0
(c) –15 (d) –12

Answer and Explanation: Using the proper signs, the above expression can be written as

$2 + 3 - 4 \times 5 \div 1$

$= 5 - 20 = -15$

Hence, option (c) is correct.

Illustration 3: Which of the following changes would make the given expression correct?

$4 + 4 \times 4 - 4 \div 4 = 4$

(a) '+' and '×' (b) '–' and '÷'
(c) '+' and '÷' (d) both 'b' and 'c'

Answer and Explanation: By making inter-changes given in 'a' we get the expression as:

$4 \times 4 + 4 - 4 \div 4 = 4$ which is false.

By making interchanges given in 'b', we get expression as

$4 + 4 \times 4 \div 4 - 4 = 4$ which is true.

By making interchanges given in option 'c', we get the expression as

$4 \div 4 \times 4 - 4 + 4 = 4$ which is also true.

Hence, option (d) is correct.

Illustration 4: If $A"B$ means add B to A, $A'B$ implies subtract B from A, $A@B$ implies divide A by B, $A*B$ means multiply A with B.

If a train travels with a speed of S_1 from point A to B and returns with a speed of S_2. The average speed will be represented using the above explained notations?

(a) $(2*S_1"S_2)/(S_1"S_2)$
(b) $(2*S_1*S_2)/(S_1"S_2)$
(c) $(2*S_1"S_2)/(S_1'S_2)$
(d) $(2*S_1@S_2)/(S_1'S_2)$

Answer and Explanation: We know that the formula for average speed is given by

$2S_1S_2/(S_1 + S_2)$

Hence the correct option is (b).

Illustration 5: If (l, m, n) is represented as $v\,[(l + 1)(m + 1)/(n + 1)]$

What is the value of (48, 63, 80)?

(a) 56/9 (b) 55/9
(c) 6 (d) 7

Answer and Explanation: Putting the value in the above equation we get

$\sqrt{[(48 + 1)(63 + 1)/(80 + 1)]}$

$= \sqrt{(49 \times 64/81)} = (7 \times 8)/9 = 56/9$

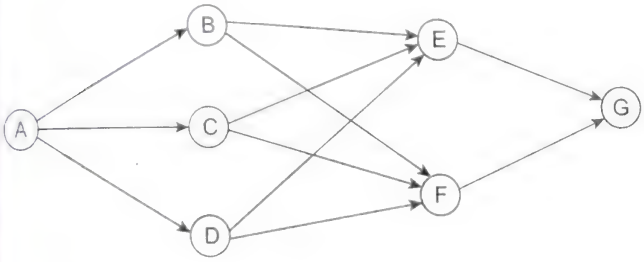
Hence, the correct option is (a).

CHAPTER 8

Routes and Network Diagrams

A look at the past trends of various MBA entrance and aptitude exams, shows that questions based on routes and network diagrams are asked very frequently in these days. Hence, there is a clear need to understand these question types and how to think inside them. The typical question on routes and network diagrams is built around a travel/transport situation between two nodes, with several nodes in between them. In such cases:

Let us look at the logic, involved with respect to the number of routes diagram. Consider the given figure where you need to travel from A to G, along the given routes and you have to find how many routes there are.



Can you count 6? ABEG, ABFG, ACEG, ACFG, ADEG and ADFG! What is the best way to think of the number of routes? How about 3 ¥ 2 = 6? What's the thought process of that, you will ask me. Well, think of it as follows. Imagine: Node A as Stage 0; Nodes B, C and D as Stage 1 (since they are symmetrical to each other) and Nodes E and F as stage 2 and Node G as stage 3.

Going from stage 0 to stage 1 (A to any of B/C/D) has 3 routes.

Further, going from stage 1 to stage 2 has 2 routes (not 6!), since if you have reached stage 1, it means that you are already at either of the three nodes of stage 1 (Viz: B/C or D). In such a case, for the movement from stage 1 to stage 2, there will be 2 routes available for you to move. i.e., if you are moving from B, you can either take BE or BF. Similarly, if you are at Node C, you can only move in two ways. Hence, there are 2 routes to go from stage 1 to stage 2.

Similarly, going from stage 2 to stage 3, you have 1 route only. This is again because, if you are at E, you would have 1 way to move forward to G. Likewise, if you are at F, you will have only 1 way to move forward to G.

Thus, your journey consists to
Stage 0 - Stage 1 - Stage 2 - Stage 3 = 3 ¥ 2 = 6 ways.

So, how do you handle an asymmetrical situation? Suppose, they say that the route from B to E is closed for repairs. How do we deal with that situation?

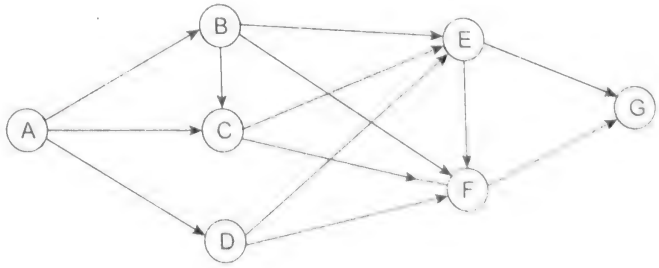
Well, the answer is to divide stage 1 into stage 1A (symmetrical nodes C/D) and stage 1B (asymmetrical node B). Your journeys from A (stage 0) to G (stage 3) can go through:

Stage 0 - Stage 1A - Stage 2 - Stage 3 = 2 ¥ 2 = 4 ways

OR Stage 0 - Stage 1B - Stage 2 - Stage 3 = 1 ¥ 1 = 1 way. (stage 0 to stage 1B is necessarily A to B; & stage 1B to stage 2 is necessarily B to F, since B to E is closed).

Thus, there are a total of 5 ways in which you can go from A to G in this case.

Once you understand this logic, you can handle high degree of asymmetry in your route networks too. Take a look at the next figure in this discussion. In this, I have made B and E as asymmetrical nodes.



In the given figure, C and D are symmetrical since both of them provide the same number of routes going forward. Hence, you do not want to disturb your symmetrical thinking through A-C or A-D. B, however, is clearly asymmetrical as well as E. Hence, in this case, divide the stages into:

Stage #	Nodes
0	A – only 1 node
1A	C,D – 2 nodes
1B	B – only 1 node
2A	E
2B	F
3	G

Alternate route structures are:

0-1A-2A-3; 0-1A-2B-3; 0-1B-2A-3; 0-1B-2B-3.

The following table will clarify the thinking required to count these.

Route Structure	Number of routes	Remarks
0-1A-2A-3	$2 \times 1 \times 2 = 4$	0 to 1A is A to C or D, hence 2 routes. 1A to 2A leads you to E – hence, 1 route from either C or D. 2A to 3 is 2 routes, since you can go EG or EFG. (Note: You could also think of this route structure as 0-1A-2A-2B-3. But, that would be only required in case there are a lot of further complications in the routes. In this case, you do not need to do it, since it is obvious that if you are at E, you can move to G directly or through F.)
0-1A-2B-3	$2 \times 1 \times 1 = 2$	0 to 1A is A to C or D, hence, 2 routes. 1A to 2B leads you to F – hence, 1 route from either C or D. 2B to 3 is 1 route.
0-1B-2A-3	$1 \times 2 \times 2 = 4$	0 to 1B is A to B, hence, 1 route. 1B to 2A, means going from the node B to E. It can be seen from the figure that there are 2 ways of doing it- BE or BCE. 2A to 3 is 2 routes, since you can go EG or EFG.
0-1B-2B-3	$1 \times 2 \times 1 = 2$	0 to 1B is A to B, hence, 1 route. 1B to 2B, means going from the node B to F. It can be seen from the figure that there are 2 ways of doing it- BF or BCF. 2B to 3 is 1 route only, since you have to go from F (if you are at 2B).
Total routes	12	For the total routes, you have to add the number of routes for each route structure.

As, I have already mentioned in the table, there could be a need of sub dividing the route structures as follows: 0-1A-2A-3; 0-1A-2A-2B-3; 0-1A-2B-3; 0-1B-2A-3; 0-1B-2A-2B-3; 0-1B-1A-2A-3 and 0-1B-1A-2A-2B-3. However, you can avoid the need to use so many route structures by thinking about it in aggregations as shown in the table, where we have seen that to go from E to G, you can go EFG or EG directly. The thing you need to understand about this, is that you use the sub-structures only if they are required to sort out the confusions. Hence, use this logic of breaking the routes into sub-structures judiciously.

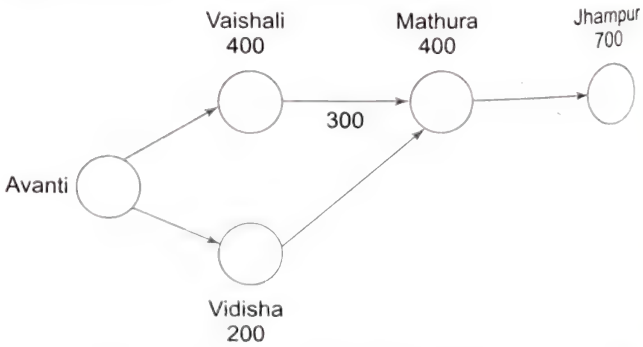
Sometimes, in questions on routes and diagrams, there is additional data about, time taken, capacity of the routes and cost for the routes. However, even in such questions, the ability to count the number of routes is critical to solving questions on routes and network diagrams.

Let us now move into a few solved examples, in order to have a look at various patterns of questions. Once, you finish looking at these solved examples, please go into the exercise where you will find a complete question bank to practice network diagram questions. Also, make sure to complete your knowledge of these questions, do go into the past year question papers of CAT (given at the end of the book) to see other examples of questions on network diagrams.

Illustration 1

Directions for Questions 1 to 3: Answer these questions based on the pipeline diagram below.

The following sketch shows the pipelines carrying material from one location to another. Each location has a demand for material. The demand at Vaishali is 400, at Mathura is 400, at Jhampur is 700 and at Vidisha is 200. Each arrow indicates the direction of material flow through the pipeline. The flow from Vaishali to Mathura is 300. The quantity of material flow is such that the demands at all these locations are exactly met. The capacity of each pipeline is 1000.



1. What is the free capacity available in the Avanti-Vidisha pipeline?
   (a) 300 (b) 200
   (c) 100 (d) 0
2. What is the free capacity available from Avanti to Vaishali?

(a) 0 (b) 100
(c) 200 (d) 300

3. The quantity moved from Avanti to Vidisha is:
(a) 200 (b) 800
(c) 700 (d) 1000

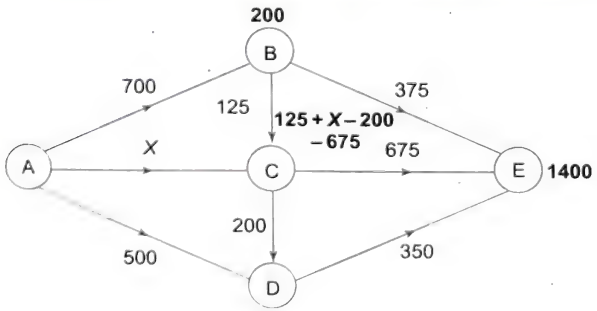
Solution Since 700 is required at Jhampur, the requirement at Mathura must be 1100, which has to be supplied from the two pipelines coming into Mathura.

It is clear that since Vaishali to Mathura is only 300, the Vidisha-Mathura pipeline should carry 800. Hence, Avanti-Vidisha should have 1000.

1. There is no free capacity in the Avanti Vidisha pipeline. Option (d) is correct.
2. Avanti-Vaishali flow should be 700 and hence, the free capacity is 300. Option (d) is correct.
3. 1000. Option (d) is correct.

Illustration 2

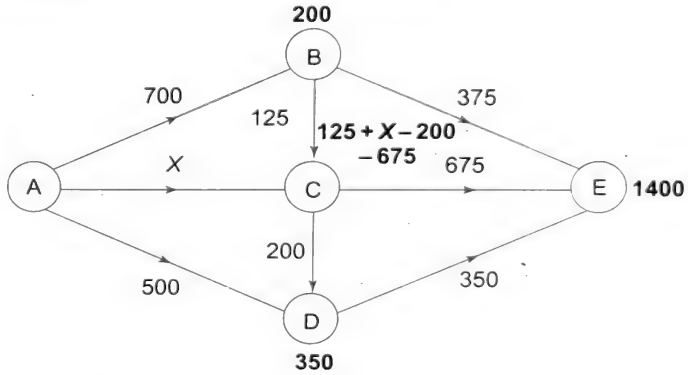
Directions for Questions 1 to 3:



In the network diagram above, the figures represent the flow of natural gas through pipelines between major cities A, B, C, D & E (in suitable units). Assume that supply equals demand in the network (although not on individual nodes).

1. What is the number of units demanded at B?
(a) 175 (b) 200
(c) 225 (d) 250
2. If the number of units demanded in C is 225, what is the value of x?
(a) 975 (b) 875
(c) 775 (d) 950
3. What is the demand in D?
(a) 300 (b) 350
(c) 375 (d) 450

Solution Refer to the following figure for the solution:



1. From the figure, it would be 700 - (375 + 125) = 200. Option (b) is correct.
2. 125 + X - 200 - 675 = 225 Æ X = 975. Option (a) is correct.
3. 500 + 200 - 350 = 350. Option (b) is correct.

CHAPTER 9

Permutations and Combinations

INTRODUCTION

Permutations and combinations has been one of the most important and regular topics that have been asked in the Civil Services exam in the past —over the last decade where the General Mental Ability was a part of the General Studies Paper, and also in 2011 when the Civil Services introduced the Civil Services Aptitude Test.

Before you move to the theory of this chapter we would like you to take a look at the following text about how you should look at this topic.

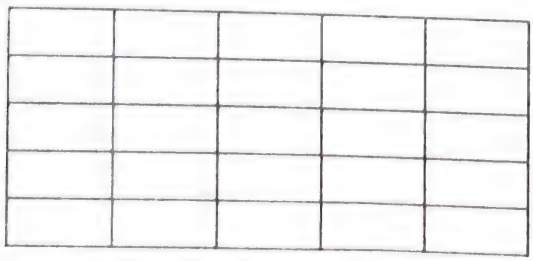
In my experience, students can be divided into two broad categories on the basis of their ability in solving this chapter.

Category 1: Students who are comfortable in solving questions on this block, since they understand the underlying concepts well.

Category 2: Students who are not able to tackle questions in this block of chapters since they are not conversant with the counting tools and methods in this block.

If you belong to the second category of students, the main thing you would need to do is to familiarise yourself with the counting methods and techniques of Permutations and Combinations. Once you are through with the same, you would find yourself relatively comfortable at both Permutations and Combinations (P & C) and Probability—the chapters in this block. However, before we start looking at these counting methods, right at the outset, I would want you to remove any negative experiences you might have had while trying to study P and C and Probability. So if you belong to the second category of students, you are advised to read on:

Look at the following table:



Suppose, I were to ask you to count the number of cells in the table above, how would you do it??

$5 \times 5 = 25$!!

I guess, all of you realise the fact that the number of cells in this table is given by the product of the number of rows and columns. However, if you ask a 5-year old child to count the same, he would be counting the number of cells physically. In fact, when you were 5 years old, you would also have required to do a physical count of the number of cells in the table. However, there must have come a point where you must have understood that in all such situations (no. of cells in a table, no. of students in a class, etc.) the total count is got by simply multiplying the number of rows and the number of columns. What I am interested in pointing out to you, is that the discovery of this process for this specific counting situation surely made your counting easier!! What used to take you much longer started taking you shorter times. Not only that, in situations where the count was too large (e.g. 100 rows $\times$ 48 columns) an infeasible count became extremely easy. So why am I telling you this??

Simply because just as the rows into columns tool for counting had the effect of making your count easier, so also the tools for counting which P and C describe will also have the effect of making counting easier for you in the specific situations of counting that you will encounter. My experience of training students shows that the only reason students have problems in this block is because it has not been explained properly to them in their +2 classes.

If you try to approach these chapters with the approach that it is not meant to complicate your life but to simplify it, you might end up finding out that there is nothing much to fear in this chapter.

THE TOOL BOX APPROACH TO P and C

While studying P and C, your primary objective should be to familiarise yourself with each and every counting situation. You also need to realise that there are a limited number of counting situations which you need to tackle.

You can look at this as the process of the creation of what can be described as a counting tool box.

Once this tool box is created, you would be able to understand the basic situations for counting. Then solving tough questions becomes a matter of simplifying the language of the question into the language of the answer—a matter I would come back to later.

Let us now proceed to list out the specific counting situations which you need to get a hold on. You need to know and understand the following twelve situations of counting and the tools that are used in these situations. Please take note that these tools are explained in details in various parts of the text on the Permutations and Combinations chapter. You are required to keep these in mind along with the specific situations in which these apply. Look at these as a kind of comprehensive list of situations in which you should know how to count using mathematical tools for your convenience.

The Twelve Counting Situations and Their Tools

Tool No. 1: The ${}^nC_r$ tool—This tool is used for the specific situation of counting the number of ways of selecting r things out of n **distinct** things.

Example: Selecting a team of 11 cricketers out of a team of 16 distinct players will be ${}^{16}C_{11}$.

Tool No. 2: The tool for counting the number of selections of r things out of n **identical** things. (Always 1)

Example: The number of ways of selecting 3 letters out of five A's. Note that in such cases there will only be one way of selecting them.

Tool No. 3: The 2^n tool—The tool for selecting any number (including 0) of things from n **distinct** things (${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$). Example the number of ways you can or cannot eat sweets at a party if there are ten sweets and you have the option of eating one piece of as many sweets as you like or even of not eating any sweet. Will be given by 2^{10}.

Corollary: The $2^n - 1$ tool—Used when zero selections are not allowed. For instance in the above situation if you are asked to eat at least one sweet and rejecting all sweets is not an option.

Tool No. 4: The $n + 1$ tool—The tool for selecting any number (including 0) of things from n **identical** things. For instance, if you have to select any number of letters from A, A, A, A and A then you can do it in six ways.

Tool No. 5: The ${}^{n+r-1}C_{r-1}$ tool—The tool for distributing n identical things amongst r people/groups such that any person/group might get any number (including 0). For instance, if you have to distribute 7 identical gifts amongst 5 children in a party then it can be done in ${}^{(7+5-1)}C_{(5-1)}$ ways. i.e. ${}^{11}C_4$ ways.

Tool No. 6: The ${}^{n-1}C_{r-1}$ tool—The tool for distributing n identical things amongst r people/groups such that any person/group might get any number (except 0).

Suppose in the above case you have to give at least one gift to each child it can be done in: ${}^6C_4$ ways.

Tool No. 7: The mnp rule tool—The tool for counting the number of ways of doing three things when each of them has to be done and there are m ways of doing the first thing, n ways of doing the second thing and p ways of doing the last thing. Suppose you have to go from Lucknow to Varanasi to Patna to Ranchi to Jamshedpur and there are 5 trains from Lucknow to Varanasi and there are 4 trains from Varanasi to Patna, six from Patna to Ranchi and 3 from Ranchi to Jamshedpur, then the number of ways of going from Lucknow to Jamshedpur through Varanasi, Patna & Ranchi is $5 \times 4 \times 6 \times 3 = 360$.

Note that this tool is extremely crucial and is used to form numbers and words.

For example how many 4 digit numbers can you form using the digits 0, 1, 2, 3, 4, 5 and 6 only without repeating any digits. In this situation the work outline is to choose a digit for the first place (= 6, any one out of 1, 2, 3, 4, 5 and 6 as zero cannot be used there), then choosing a digit for the second place (= 6, select any one out of 5 remaining digits plus 0), then a digit for the third place (5 ways) and a digit for the fourth place (4 ways).

Tool No 8: The $r!$ tool for arrangement—This tool is used for counting the number of ways in which you can arrange r things in r places. Notice that this can be derived out of the mnp rule tool.

Tool No 9: The AND rule tool—Whenever you describe the counting situation and connect two different parts of the count by using the conjunction **'AND'**, you will always replace the **'AND'** with a multiplication sign between the two parts of the count. This is also used for solving Probability questions. For instance, suppose you have to choose a vowel **'AND'** a consonant from the letters of the word PERMIT, you can do it in ${}^4C_1 \times {}^2C_1$ ways.

Tool No 10: The OR rule tool—Whenever you describe the counting situation and connect two different parts of the count by using the conjunction **'OR'** you will always replace the **'OR'** with an Addition sign between the two parts of the count. This is also used for solving Probability questions. For instance, suppose you have to select either 2 vowels or a consonant from the letters of the word PERMIT, you can do it in ${}^4C_2 + {}^2C_1$ ways.

Tool No 11: The Circular Permutation Tool [$(n-1)!$ Tool]—This gives the number of ways in which n distinct things can be placed around a circle. The need to reduce the value of the factorial by 1 is due to the fact that in a circle there is no defined starting point.

Tool No 12: The Circular Permutation Tool when there is no distinction between clockwise and anti clockwise arrangements [$(n-1)!/2$ Tool]—This gives the number of ways in which n distinct things can be placed around a circle when there is no distinction between clockwise and anti clockwise arrangements. In such a case the number of circular permutations are just divided by 2.

Two More Issues

(1) What about the ${}^nP_r$ tool? The ${}^nP_r$ tool is used to arrange r things amongst n. However, my experience shows that the introduction of this tool just adds to the confusion for most students. A little bit of smart thinking gets rid of all the problems that the ${}^nP_r$ tool creates in your mind. For this let us consider the difference between the ${}^nP_r$ and the ${}^nC_r$ tools.

${}^nC_r = n!/[r! \leftrightarrow (n-r)!]$ while ${}^nP_r = n!/[(n-r)!]$.

A closer look would reveal that the relationship between the ${}^nP_r$ and the ${}^nC_r$ tools can be summarised as:

${}^nP_r = {}^nC_r \times r!$

Read this as: If you have to arrange r things amongst n things, then simply select r things amongst n things and use the $r!$ tool for the arrangement of the selected r items amongst themselves. This will result in the same answer as the ${}^nP_r$ tool.

In fact, for all questions involving arrangements always solve the question in two parts—First finish the selection within the question and then arrange the selected items amongst themselves. This will remove a lot of confusions in questions of this chapter.

(2) Treat Both Permutations and Combinations and Probability as English language chapters rather than Maths chapters: One of the key discoveries in getting stronger at this block is that after knowing the basic twelve counting situations and their tools (as explained above) you need to stop treating the questions in this chapter as questions in Mathematics and rather treat them as questions in English. Thus, while solving a question in these chapters your main concentration and effort should be on converting the question language into the answer language.

What do I mean by this?

Consider this:

Question Language: What is the number of ways of forming 4 letter words from 5 vowels and 6 consonants such that the word contains one vowel and three consonants?

To solve this question, first create the answer language in your mind:

Answer Language: Select one vowel out of 5 **AND** select three consonants out of 6 **AND** arrange the four selected letters amongst themselves.

Once you have this language the remaining part of the answer is just a matter of using the correct tools.

Thus, the answer is ${}^5C_1 \times {}^6C_3 \times 4!$

As you can see, the main issue in getting the solution to this question was working out the language by connecting the various parts of the solution using the conjunction AND (In this case, the conjunction OR was not required.). Once you had the language all you had to do was use the correct tools to count.

The most difficult of questions are solved this way. This is why I advise you to treat this chapter as a language chapter and not as a chapter in Mathematics!!

Standard Theory:

Factorial Notation! Or $\lfloor$

$\lfloor \underline{n} = n(n-1)(n-2)\ldots 3.2.1$

$n! = \underline{n}(n-1)(n-2)\ldots 3.2.1$

= Product of n consecutive integers starting from 1.

1. $0! = 1$
2. Factorials of only Natural numbers are defined.
   $n!$ is defined only for $n \geq 0$
   $n!$ is not defined for $n < 0$
4. ${}^nC_r = 1$ when $n = r$.
5. Combinations (represented by ${}^nC_r$) can be defined as the number of ways in which r things at a time can be **SELECTED** from amongst n things available for selection.
   The key word here is **SELECTION**. Please understand here that the order in which the r things are selected has no importance in the counting of combinations.
   ${}^nC_r$ = Number of combinations (selections) of n things taken r at a time.
   ${}^nC_r = n!/[r!\,(n-r)!]$; where $n \geq r$ (n is greater than or equal to r).

Some typical situations where selection / combination is used:

a) Selection of people for a team, a party, a job, an office etc. (e.g. Selection of a cricket team of 11 from 16 members)

b) Selection of a set of objects (like letters, hats, points pants, shirts, etc) from amongst another set available for selection.

In other words any selection in which the order of selection holds no importance is counted by using combinations.

6. Permutations (represented by ${}^nP_r$) can be defined as the number of ways in which r things at a time can be **SELECTED & ARRANGED** at a time from amongst n things.
   The key word here is **ARRANGEMENT**. Hence please understand here that the order in which the

r things are arranged has critical importance in the counting of permutations.

In other words permutations can also be referred to as an **ORDERED SELECTION**.

${}^nP_r$ = number of permutations (arrangements) of n things taken r at a time.

${}^nP_r = n!/(n-r)!; n \geq r$

Some typical situations where **ordered selection / permutations** are used:

(a) Making words and numbers from a set of available letters and digits respectively

(b) Filling posts with people

(c) Selection of batting order of a cricket team of 11 from 16 members

(d) Putting distinct objects/people in distinct places, e.g. making people sit, putting letters in envelopes, finishing order in horse race, etc.)

The exact difference between selection and arrangement can be seen through the illustration below:

SELECTION

Suppose we have three men A, B and C out of which 2 men have to be selected to two posts.

This can be done in the following ways: AB, AC or BC (These three represent the basic selections of 2 people out of three which are possible. Physically they can be counted as 3 distinct selections. This value can also be got by using ${}^3C_2$. Note here that we are counting AB and BA as one single selection. So also AC and CA and BC and CB are considered to be the same instances of selection since the order of selection is not important.

ARRANGEMENT

Suppose we have three men A, B and C out of which 2 men have to be selected to the post of captain and vice captain of a team.

In this case we have to take AB and BA as two different instances since the order of the arrangement makes a difference in who is the captain and who is the vice captain.

Similarly, we have BC and CB and AC and CA as 4 more instances. Thus in all there could be 6 arrangements of 2 things out of three.

This is given by ${}^3P_2 = 6$.

7. **The Relationship Between Permutation & Combination:**

   When we look at the formulae for Permutations and Combinations and compare the two we see that,

   ${}^nP_r = r! \times {}^nC_r$

   $= {}^nC_r \times {}^rP_r$

   This in words can be said as:

   The permutation or arrangement of r things out of n is nothing but the selection of r things out of n followed by the arrangement of the r selected things amongst themselves.

8. **MNP Rule:** If there are three things to do and there are M ways of doing the first thing, N ways of doing the second thing and P ways of doing the third thing then there will be $\mathbf{M \times N \times P}$ ways of doing all the three things together.The works are mutually inclusive.

   This is used to for situations like:

   The numbers 1, 2, 3, 4 and 5 are to be used for forming 3 digit numbers without repetition. In how many ways can this be done?

   Using the MNP rule you can visualise this as: There are three things to do→ The first digit can be selected in 5 distinct ways, the second can be selected in 4 ways and the third can be selected in 3 different ways. Hence, the total number of 3 digit numbers that can be formed are $5 \times 4 \times 3 = 60$

9. When the pieces of work are mutually exclusive,there are M+N+P ways of doing the complete work.

Important Results

The following results are important as they help in problem solving.

1. Number of permutations (or arrangements) of n different things taken all at a time = $n!$
2. Number of permutations of n things out of which P_1 are alike and are of one type, P_2 are alike and are of a second type and P_3 are alike and are of a third type and the rest are all different = $n! / P_1! P_2! P_3!$

   Illustration: The number of words formed with the letters of the word Allahabad.

   Solution: Total number of Letters = 9 of which A occurs four times, L occurs twice and the rest are all different.

   Total number of words formed = 9! / (4! 2! 1!)
3. Number of permutations of n different things taken r at a time when repetition is allowed = $n \times n \times n \times \ldots$ (r times) = n^r.

   Illustration: In how many ways can 4 rings be worn in the index, ring finger and middle finger if there is no restriction of the number of rings to be worn on any finger?

   Solution: Each of the 4 rings could be worn in 3 ways either on the index, ring or middle finger.

   So, four rings could be worn in $3\times3\times3\times3 = 3^4$ ways.
4. Number of selections of r things out of n identical things = 1

   Illustration: In how many ways 5 marbles can be chosen out of 100 identical marbles?

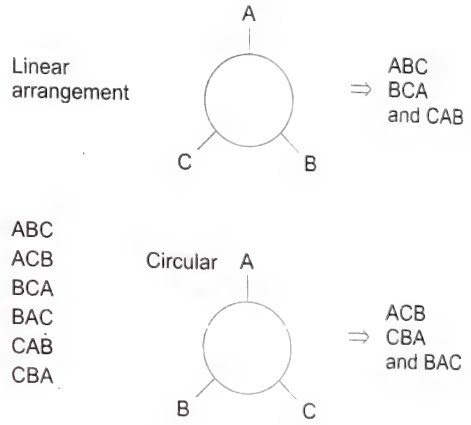
   Solution: Since, all the 100 marbles are identical

   Hence, Number of ways to select 5 marbles = 1

5. Total number of selections of zero or more things out of k identical things = k + 1.
   This includes the case when zero articles are selected.
6. Total number of selections of zero or more things out of n different things =
   ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$
   ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$
   Corollary: The number of selections of 1 or more things out of *n* different things = ${}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$
   $= 2^n - 1$
7. Number of ways of distributing *n* identical things among *r* persons when each person may get any number of things = ${}^{n+r-1}C_{r-1}$
   Imagine a situation where 27 marbles have to be distributed amongst 4 people such that each one of them can get any number of marbles (including zero marbles). Then for this situation we have, n = 27(no. of identical objects), *r* = 4 (no. of people) and the answer of the number of ways this can be achieved is given by:
   ${}^{n+r-1}C_{r-1} = {}^{30}C_3$.
8. Corollary: No. of ways of dividing *n* non distinct things to *r* distinct groups are:
   ${}^{n-1}C_{r-1} \rightarrow$ For non-empty groups only
   Also, the number of ways in which *n* distinct things can be distributed to *r* different persons:
   $= r^n$
9. Number of ways of dividing m + n different things in two groups containing *m* and *n* things respectively = ${}^{m+n}C_n \times {}^mC_m =$
   $= (m + n)!/m!\ n!$
   Or, ${}^{m+n}C_m \times {}^nC_n = (m+n)!/n!\ m!$
10. Number of ways of dividing 2n different things in two groups containing *n* things = 2n! / n! n! 2!
11. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
12. ${}^nC_x = {}^nC_y \Rightarrow x = y$ or $x + y = n$
13. ${}^nC_r = {}^nC_{n-r}$
14. $r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}$
15. ${}^nC_r/(r+1) = {}^{n+1}C_{r+1}/(n+1)$
16. For ${}^nC_r$ to be greatest,
    a. if n is even, r = n/2
    b. if n is odd, r = (n + 1)/2 or (n − 1)/2
17. Number of selections of *r* things out of *n* different things
    a. When *k* particular things are always included = ${}^{n-k}C_{r-k}$
    b. When *k* particular things are excluded = ${}^{n-k}C_r$
    c. When all the *k* particular things are not together in any selection
       $= {}^nC_r - {}^{n-k}C_{r-k}$
       No. of ways of doing a work with given restriction = total no. of ways of doing it — no. of ways of doing the same work with opposite restriction.
18. The total number of ways in which 0 to *n* things can be selected out of *n* things such that *p* are of one type, *q* are of another type and the balance *r* of different types is given by: $(p + 1)(q + 1)(2^r - 1)$.
19. Total number of ways of taking some or all out of p+q+r things such that p are of one type and q are of another type and r of a third type
    $= (p + 1)(q + 1)(r + 1) - 1$ [Only non-empty sets]
20. $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$
21. Number of selections of *k* consecutive things out of *n* things in a row = n − k + 1

CIRCULAR PERMUTATIONS

Consider two situations:
There are three A, B, C. In the first case, they are arranged linearly and in the other, around a circular table –



For the linear arrangement, each arrangement is a totally new way. For circular arrangements, three linear arrangements are represented by one and the same circular arrangement. So, for six linear arrangements, there correspond only 2 circular arrangements. This happens because there is no concept of a starting point on a circular arrangement. (i.e., the starting point is not defined.)
Generalising the whole process, for n!, there corresponds to be (1 /n) n! ways.

Important Results

1. Number of ways of arranging n people on a circular track (circular arrangement) = (n − 1)!
2. When clockwise and anti-clockwise observation are not different then number of circular arrangements of *n* different things = (n − 1)! /2

e.g. the case of a necklace with different beads, the same arrangement when looked at from the opposite side becomes anti-clockwise.

3. Number of selection of k consecutive things out of n things in a circle
   $= n$ when $k < n$
   $= 1$ when $k = n$

Some more results

1. Number of terms in $(a_1 + a_2 + \ldots + a_n)^m$ is ${}^{m+n-1}C_{n-1}$
   Illustration: Find the number of terms in $(a+b+c)^2$.
   Solution: $n = 3$, $m = 2$
   ${}^{m+n-1}C_{n-1} = {}^4C_2 = 6$
   Corollary: Number of terms in
   $(1 + x + x^2 + \ldots . x^n)^m$ is $mn+1$
2. Number of zeroes ending the number represented by
   $n! = [n/5]+[n/5^2]+[n/5^3]+\ldots[n/5^x]$
   [] Shows greatest integer function where $5^x \le n$
   Illustration: Find the number of zeroes at the end of 1000!
   Solution: $[1000/5] + [1000/5^2] + [1000/5^3] + [1000/5^4]$
   $200 + 40 + 8 + 1 = 249$
   Corollary: Exponent of 3 in $n! = [n!/3] + [n!/32] + [n!/3^3] + \ldots[n!/3^x]$ where $3^x \le n$
   [] Shows greatest integer fn.
   Illustration: Find how many exponents of 3 will be there in 24!.
   Solution: $[24/3] + [24/32] = 8 + 2 = 10$
3. Number of squares in a square of $n \times n$ side $= 1^2 + 2^2 + 3^2 + 4^2 + \ldots . n^2$
   Number of rectangles in a squares of $n \times n$ side $= 1^3 + 2^3 + 3^3 + 4^3 + \ldots . n^3$. (This includes the number of squares.)
   Thus the number of squares and rectangles in the following figure are given by:

Number of squares $= 1^2 + 2^2 + 3^2 = 14 = \in n^2$
Number of rectangles $= 1^3 + 2^3 + 3^3 = 36\ (\in n)^2 = \in n^3$ for the rectangle.
A rectangle having m rows and n columns:
The number of squares is given by: $m.n + (m-1)(n-1) + (m-2)(n-2) + \ldots .$ until any of $(m-x)$ or $(n-x)$ comes to 1.
The number of rectangles is given by: $(1 + 2 + \ldots . + m)(1 + 2 + \ldots . + n)$

SOLVED EXAMPLES

In the following examples the solution is given upto the point of writing down the formula that will apply for the particular question. The student is expected to calculate the values after understanding the solution.

1. Find the number of permutations of 6 things taken 4 at a time.

Sol. The answer will be given by ${}^6P_4$.

2. How many 3-digit numbers can be formed out of the digits 1, 2, 3, 4 and 5.

Sol. Forming numbers requires an ordered selection. Hence, the answer will be ${}^5P_3$.

3. In how may ways can the 7 letters M, N, O, P, Q, R, S be arranged so that P and Q occupy continuous positions?

Sol. For arranging the 7 letters keeping P and Q always together we have to view P and Q as one letter. Let this be denoted by PQ.
Then, we have to arrange the letters M, N, O, PQ, R and S in a linear arrangement. Here, it is like arranging 6 letters in 6 ways (since 2 letters are counted as one). This can be done in 6! ways.
However, the solution is not complete at this point of time since in the count of 6! the internal arrangement between P and Q is neglected. This can be done in 2! ways. Hence, the required answer is $6! \times 2!$.
Task for the student: What would happen if the letters P, Q and R are to be together? (Ans: $5! \times 3!$)
What if P and Q are never together? (Answer will be given by the formula: Total number of ways – Number of ways they are always together)

4. Of the different words that can be formed from the letters of the words BEGINS how many begin with B and end with S?

Sol. B & S are fixed at the start and the end positions. Hence, we have to arrange E, G, I and N amongst themselves. This can be done in 4! ways.
Task for the student: What will be the number of words that can be formed with the letters of the word BEGINS which have B and S at the extreme positions? (Answer $= 4! \times 2!$)

5. In how many ways can the letters of the word VALEDICTORY be arranged, so that all the vowels are adjacent to each other?

Sol. There are 4 vowels and 7 consonants in 'valedictory'. If these have to be kept together, we have to consider AEIO as one letter. Then the problem transforms itself into arranging 8 letters amongst themselves (8! ways). Besides, we have to look at the internal arrangement of the 4 vowels amongst themselves. (4! ways)
Hence Answer $= 8! \times 4!$.

6. If there are two kinds of hats, red and blue and at least 5 of each kind in how many ways can the hats be put in each of 5 different boxes?

Sol: The significance of at least 5 hats of each kind is that while putting a hat in each box, we have the option of putting either a red or a blue hat. (If this was not given, there would have been an uncertainty in the number of possibilities of putting a hat in a box.)
Thus in this question for every task of putting a hat in a box we have the possibility of either putting a red hat or a blue hat. The solution can then be looked at as: there are 5 tasks each of which can be done in 2 ways. Through the MNP rule we have the total number of ways = 2^5 (Answer).

7. In how many ways can 4 Indians and 4 Nepalese people be seated around a round table so that no two Indians are in adjacent positions?

Sol. If we first put 4 Indians around the round table, we can do this in 3! ways.
Once the 4 Indians are placed around the round table, we have to place the four Nepalese around the same round table. Now, since the Indians are already placed we can do this in 4! ways (as the starting point is defined when we put the Indians. Try to visualise this around a circle for placing 2 Indians and 2 Nepalese.)
Hence, Answer = $3! \times 4!$

8. How many numbers greater than a million can be formed from the digits 1, 2, 3, 0, 4, 2, 3?

Sol. In order to form a number greater than a million we should have a 7 digit number. Since we have only seven digits with us we cannot take 0 in the starting position. View this as 7 positions to fill:

_ _ _ _ _ _ _

To solve this question we first assume that the digits are all different. Then the first position can be filled in 6 ways (0 cannot be taken), the second in 6 ways (one of the 6 digits available for the first position was selected. Hence, we have 5 of those 6 digits available. Besides, we also have the zero as an additional digit), the third in 5 ways (6 available for the 2nd position – 1 taken for the second position.) and so on. Mathematically this can be written as:
$6 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6 \times 6!$
This would have been the answer had all the digits been distinct. But in this particular example we have two 2's and two 3's which are identical to each other. This complication is resolved as follows to get the answer:

$$\frac{6 \times 6}{2! \times 2!}$$

9. If there are 11 players to be selected from a team of 16, in how many ways can this be done?

Sol. $^{16}C_{11}$.

10. In how many ways can 18 identical white and 16 identical black balls be arranged in a row so that no two black balls are together?

Sol. When 18 identical white balls are put in a straight line, there will be 19 spaces created. Thus 16 black balls will have 19 places to fill in. This will give an answer of: $^{19}C_{16}$. (Since, the balls are identical the arrangement is not important.)

11. A mother with 7 children takes three at a time to a cinema. She goes with every group of three that she can form. How many times can she go to the cinema with distinct groups of three children and how many times each child goes to the cinema?

Sol. She will be able to do this as many times as she can form a set of three distinct children from amongst the seven children. This essentially means that the answer is the number of selections of 3 people out of 7 that can be done.
Hence, Answer = $^7C_3$.
Now, each child will go to cinema same number of time. Now fix a specific student to goto cinema same number of time. Now we have to choose 2 chidren out of 6 children.

12. For the question above, how many times will an individual child go to the cinema with her before a group is repeated?

Sol. This can be viewed as: The child for whom we are trying to calculate the number of ways is already selected. Then, we have to select 2 more children from amongst the remaining 6 to complete the group. This can be done in $^6C_2$ ways.

13. How many different sums can be formed with the following coins:
5 rupee, 1 rupee, 50 paisa, 25 paisa, 10 paisa and 1 paisa?

Sol. A distinct sum will be formed by selecting either 1 or 2 or 3 or 4 or 5 or all 6 coins.
But from the formula we have the answer to this as: $2^6 - 1$.
[Task for the student: How many different sums can be formed with the following coins:
5 rupee, 1 rupee, 50 paisa, 25 paisa, 10 paisa, 3 paisa, 2 paisa and 1 paisa?
Hint: You will have to subtract some values for double counted sums.]

14. A train is going from Bombay to Poona and makes 5 stops on the way. 3 persons enter the train during the journey with 3 different tickets. How many different sets of tickets may they have had?

Sol. Since the 3 persons are entering during the journey they could have entered at the:

1st station (from where they could have bought tickets for the 2nd, 3rd, 4th or 5th stations or for Poona→ total of 5 tickets.)

2nd station (from where they could have bought tickets for the 3rd, 4th or 5th stations or for Poona→ total of 4 tickets.)

3rd station (from where they could have bought tickets for the 4th or 5th stations or for Poona → total of 3 tickets.)

4th station (from where they could have bought tickets for the 5th station or for Poona → total of 2 tickets.)

5th station (from where they could have bought a ticket for Poona → total of 1 ticket.)

Thus, we can see that there are a total of 5+4+3+2+1= 15 tickets available out of which 3 tickets were selected. This can be done in ${}^{15}C_3$ ways (Answer).

15. Find the number of diagonals and triangles formed in a decagon.

Sol. A decagon has 10 vertices. A line is formed by selecting any two of the ten vertices. This can be done in ${}^{10}C_2$ ways. However, these ${}^{10}C_2$ lines also count the sides of the decagon.

Thus, the number of diagonals in a decagon is given by: ${}^{10}C_2 - 10$ (Answer)

Triangles are formed by selecting any three of the ten vertices of the decagon. This can be done in ${}^{10}C_3$ ways (Answer).

16. Out of 18 points in a plane, no three are in a straight line except 5 which are collinear. How many straight lines can be formed?

Sol. If all 18 points were non-collinear then the answer would have been ${}^{18}C_2$. However, in this case ${}^{18}C_2$ has double counting since the 5 collinear points are also amongst the 18. These would have been counted as ${}^{5}C_2$ whereas they should have been counted as 1. Thus, to remove the double counting and get the correct answer we need to adjust by reducing the count by $({}^{5}C_2 - 1)$.

Hence, Answer = ${}^{18}C_2 - ({}^{5}C_2 - 1) = {}^{18}C_2 - {}^{5}C_2 + 1$

17. For the above situation, how many triangles can be formed?

Sol. The triangles will be given by ${}^{18}C_3 - {}^{5}C_3$.

CHAPTER 1

HCF and LCM of Numbers

1.1 FACTORS

In a division, if a number f divides a number M completely (exactly) or in other words, if M is exactly divisible by f, then 'f' is the factor of M.

EXAMPLE: 5 divides 35 completely, so, 5 is a factor of 35.

Similarly, 2, 3, 4, 6 are all factors of 12, because each of the numbers 2, 3, 4, and 6 will divide 12 completely or, in other words 12 is divisible by 2, 3, 4 and 6.

1.2 MULTIPLES

From the above concept, if f is a factor of M, then M is a multiple of f.

EXAMPLE: 63 is completely divisible by 7, 3, 9, 21. So, 63 is a multiple of 7 or 3 or 9 or 21.

1.3 PRINCIPLE OF PRIME FACTORISATION

Any natural number (>1) is either prime or non-prime (composite).

The principle of prime factorisation states:
Each non-prime (composite) number can be uniquely broken (reduced) into two or more prime numbers (prime factors). In other words, each non-prime number is divisible by any of the prime numbers.

With the use of this principle, a non-prime number is broken into its prime factor by dividing it with different prime numbers. This is known as division method of factorisation of a number. The same is explained in the following example.

EXAMPLE: Resolve 20570 into its prime factors.

Division by prime number ↓		
	2	20570
	5	10285 → 1st Quotient
Prime Factors	11	2057 → 2nd Quotient
	11	187 → 3rd Quotient
	17	17 → 4th Quotient
		1

Thus, $20570 = 2 \times 5 \times 11 \times 11 \times 17$.

Hence, if the number is even, the division should start with 2; otherwise, rest of the prime numbers should be tried in succession.

1.4 HIGHEST COMMON FACTOR (HCF)

If two or more numbers are broken into their prime factors, then the product of the maximum common prime factors in the given numbers is the HCF of the numbers.

In other words, the HCF of two or more numbers is the greatest number (divisor) that divides all the given numbers exactly. So, HCF is also called the Greatest Common Divisor (GCD).

EXAMPLE: Find the HCF of 72, 60, 96.

Here, we first find the prime factors of each given number.

72		60		96
2 | 72	2 | 60		2 | 96
2 | 36	2 | 30		2 | 48
2 | 18	3 | 15		2 | 24
3 | 9	5		2 | 12
3			2 | 6
			3

Here $72 = ② \times ② \times 2 \times ③ \times 3$

$60 = ② \times ② \times ③ \times 5$

$96 = ② \times ② \times 2 \times 2 \times 2 \times ③$

and so HCF = product of maximum common prime factors $= 2 \times 2 \times 3 = 12$

Note: The common factors in the given numbers have been encircled.

1.5 LOWEST COMMON MULTIPLE (LCM)

The LCM of two or more than two numbers is the product of the highest powers of all the prime factors that occur in these numbers.

EXAMPLE: Find the LCM of 36, 48, 64, and 72

2	36,	48,	64,	72
2	18,	24,	32,	36
2	9,	12,	16,	18
2	9,	6,	8,	9
3	9,	3,	4,	9
3	3,	1,	4,	3
	1,	1,	4,	1

$\therefore$ LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 4 = 576$.

1.6 PRODUCT OF TWO NUMBERS

HCF of numbers × LCM of numbers = Product of numbers

i.e., if the numbers are A and B, then

HCF of A and B × LCM of A and $B = A \times B$

1.7 DIFFERENCE BETWEEN HCF AND LCM

HCF of x, y and z	LCM of x, y and z
is the **Highest Divisor** which can exactly divide x, y and z.	is the **Least Dividend** which is exactly divisible by x, y and z.

SOLVED EXAMPLES

CHAPTER 2

Fractions

2.1 DEFINITION

A number of the type $\frac{x}{y}$ which represents x number of parts out of y number of equal parts of a thing is called a **fraction.**

$\therefore$ fraction $\frac{2}{7}$ represents 2 equal parts out of 7 equal parts of a thing. In the figure given below, the shaded part represents $\frac{2}{7}$

$\frac{2}{7}$

$\therefore$ Fraction $= \frac{\text{Numerator}}{\text{Denominator}}$.

Such a fraction is known as common fraction or vulgar fraction

- A fraction, whose denominator is 10, 100 or 1000, etc., is called a **decimal fraction**.
- Fractions whose denominators are same, are called **like fractions**, e.g., $\frac{3}{8}, \frac{5}{8}$ are like fractions.
- Fractions whose denominators are different, are called **unlike fractions**, e.g. $\frac{3}{4}, \frac{9}{11}$ are unlike fractions.

2.2 COMPARISON OF FRACTIONS

Two or more different fractions can be compared with the help of the following rules:

Rule 1 (for same denominators)

When two fractions have the same denominator, the greater fraction is the one which has the greater numerator.

EXAMPLE: Thus, $\frac{5}{11}$ is greater than $\frac{3}{11}$

Rule 2 (for same numerators)

When two fractions have the same numerator, the greater fraction is the one which has the smaller denominator.

EXAMPLE: Thus, $\frac{7}{13}$ is greater than $\frac{7}{19}$

Rule 3

When two or more fractions with different denominators and different numerators are to be compared, then the following simple technique is to be used:

Step 1 Among all the given fractions,
let the maximum number of digits in the numerator $= n$
the maximum number of digits in the denominator $= d$

Step 2 Find $(d - n)$.

Step 3 If $(d - n) = 0$ or 1, multiply each fraction by 10. If $(d - n) = 2, 3, 4 \ldots$ multiply each given fraction by 10^2, 10^3, $10^4 \ldots$ respectively.

Step 4 After multiplication, find only the integer value of the resultant fraction.

Step 5 If in step 4, any of the two fractions have the same integer value, then find the next decimal place and so on.

Step 6 Compare the integer/decimal values obtained in step 4 or step 5. The fraction having the maximum value is the greatest fraction.

Note: In order to write the given fraction in ascending order the smallest fraction is written first, then the next greater one and so on. In order to write the given fraction in descending order, the greatest fraction is written first, then the next smaller and so on.

EXAMPLE: Arrange $\frac{7}{13}, \frac{493}{971}, \frac{87}{165}, \frac{123}{235}$ in descending order.

SOLUTION

Here, maximum no. of digits in the numerator n = 3 (in 493 or in 123) and maximum no. of digits in the denominator = d = 3 (in 971 or in 235)

Now, $d - n = 3 - 3 = 0$

So, multiply the given fractions by 10.

$$\underset{5}{\frac{7}{13}\times 10},\quad \underset{5}{\frac{493}{971}\times 10},\quad \underset{5}{\frac{87}{165}\times 10},\quad \underset{5}{\frac{123}{235}\times 10}$$

(integer values)

Since the integer values are same, so find the next decimal digit for these fractions.

5.3 5.0 5.2 5.2

(5.2 and 5.2 — same)

Since the value of two fractions are same, find the second decimal digit for these two fractions only, by dividing further

$$\text{Now, } \underset{5.3}{5.3} > \underset{5.27}{5.0} > \underset{5.23}{5.27} > \underset{5.0}{5.23}$$

$$\Rightarrow \frac{7}{13} > \frac{87}{165} > \frac{123}{235} > \frac{493}{971}$$

(in descending order)

2.3 FRACTIONAL PART OF A NUMBER

Fractional part of a number (or quantity) is simply the product of the related fraction and the given number.

EXAMPLE: Consider a given number as 60, then two-thirds of $60 = \frac{2}{3} \times 60 = 40$

$\therefore$ $\frac{2}{3}$ of 60 is 40 (fractional part).

> Fractional part of any number = number × its related fraction. (1)

2.2.1 Different Fractional Parts of the Same Number

Consider any number, say, 36

then $\frac{3}{4}$ of 36 = 27 (fractional part of 36)

$\frac{1}{9}$ of 36 = 4 (fractional part of 36)

$\frac{2}{3}$ of 36 = 24 (fractional part of 36)

From this, we find that as the fraction changes the fractional part of the same number also changes.

In our earlier examples, we find that

$$\frac{27}{3/4} = \frac{4}{1/9} = \frac{24}{2/3} \ldots = 36 \text{ (Fixed)}$$

(36 → Original number)

In such cases, equation (1) can be re-written as

$$\frac{\text{Any fractional part}}{\text{its related fraction}} = \text{Original number} \quad \ldots (2)$$

(Original number → it is fixed.)

EXAMPLE: A man travels $\frac{1}{4}$ part by scooter, $\frac{3}{8}$ by car, and rest 48 km by bus. Find the total distance covered.

SOLUTION

Here, total distance (i.e., original quantity) is to be found out.

$$\text{Fraction related to rest 48 km} = 1 - \underset{\downarrow}{\left(\frac{1}{4}+\frac{3}{8}\right)}$$

related fraction to total distance

Using the relation 2

$$\frac{\text{Any fractional part}}{\text{its related fraction}} = \text{Original number,}$$

Here, we find,

$$\frac{\text{Rest distance}}{\text{its related fraction}} = \text{total distance}$$

$$\Rightarrow \frac{48}{1-\left(\frac{1}{4}+\frac{3}{8}\right)} = \text{total distance}$$

$$\Rightarrow \text{total distance} = \frac{48}{3/8} = 128 \text{ km}$$

EXAMPLE: If $\frac{3}{17}$ of a number is 18, then find its two-third.

SOLUTION

Let $\frac{2}{3}$ of number = x.

Since both the fractions $\frac{3}{17}$ and $\frac{2}{3}$ are to be found out for the same number, the relation (2) can be used as*

$$\frac{\text{Fractional part}}{\text{its related fraction}} = \frac{\text{Another fractional part}}{\text{its related fraction}}$$

= original number (always)

$$\Rightarrow \frac{18}{3/17} = \frac{x}{2/3}$$

$$x = 68$$

$\therefore$ two-third of the number is 68.

Note: This relation can be used to find another fractional part directly without finding the original number.

2.4 TO FIND THE FRACTION RELATED TO BALANCE (REST) AMOUNT

Conventionally, we have learnt that fraction related to balance (rest) part is

1 – (sum of all other fractions)

It is used when **all fractions are independent.** Following example will illustrate the fact.

EXAMPLE: A person spends $\frac{3}{8}$ part of his salary on food, $\frac{1}{12}$ part of his salary on education, $\frac{1}{4}$ part of his salary on clothing. He is now left with ₹ 550. Find his total salary.

SOLUTION

Here, the spending on each item is independent, because each fraction has been indicated as out of total salary (original number).

$\therefore$ fraction related to rest part $= 1 - \left(\frac{3}{8} + \frac{1}{12} + \frac{1}{4}\right)$

$$= \frac{5}{24}$$

$\therefore$ total salary $= \dfrac{\text{Rest amount}}{\text{fraction related to rest part}}$

[Refer relation 2 of sec. 2.2.1]

$$= \frac{550}{5/24} = \text{Rs } 2640$$

Hence, for **independent fractions.**

Fraction for balance (rest) part = 1 – (sum of all independent fractions) (3)

Now, consider another example,

EXAMPLE: A person spends $\frac{3}{8}$th part of his salary on food, $\frac{1}{12}$th of the rest part on education and $\frac{1}{4}$th of the remainder on clothing. He is now left with Rs 550. Find his total salary.

SOLUTION

Here, spending on the second item (i.e., education) depends on the amount left after spending on the first item (i.e., food). Similarly, spending on the third item (i.e., clothing) depends on the amount left (remaining) after spending on the first item and the second item.

Here, spending on each item (except the first item) depends on the amount remaining, after spending on the previous item.

In such cases, all fractions (except the first one) are dependent on the previous fractions.

For **dependent fractions,**

Fraction for balance (rest) part = (1 – first fraction) × (1 – second fraction) (4)

So, in our example, using equation 4 we get,

fraction for balance (rest) part

$$= \left(1 - \frac{3}{8}\right)\left(1 - \frac{1}{12}\right)\left(1 - \frac{1}{4}\right)$$

$$= \frac{5}{8} \times \frac{11}{12} \times \frac{3}{4}$$

$$= \frac{55}{128}$$

$\therefore$ Total salary $= \dfrac{\text{Rest amount}}{\text{Fraction related to rest part}}$

$$= \frac{550}{55/128} = \text{Rs } 1280$$

Note: Observe the difference in the language of the two examples under 2.4

2.5 TO INSERT ANY NUMBER OF FRACTIONS IN BETWEEN TWO GIVEN FRACTIONS

Let two given fractions be $\frac{a}{b}$ and $\frac{x}{y}$. To insert a fraction lying between $\frac{a}{b}$ and $\frac{x}{y}$, the following steps are taken.

Step 1 The numerators of two given fractions are added to get the numerator of the **result** fraction, i.e., numerator of the result fraction $= a + x$.

Step 2 The denominators are also added to get denominator of the **result** fraction. That is, denominator of the result fraction $= b + y$

Step 3 Result fraction $= \dfrac{a + x}{b + y}$

Hence, the result fraction so obtained has its magnitude (value) lying between the two given fractions. By this method, any number of fractions can be inserted between two given fractions.

CHAPTER 3

Percentage

3.1 INTRODUCTION

The word percent means per hundred. Thus, 19 percent means, 19 parts out of 100 parts. This can also be written as $\frac{19}{100}$.

Therefore, percent is a fraction whose denominator is 100, and the numerator of this fraction is called the **Rate percent**. So, $\frac{19}{100}$ = 19 percent. Here, 19 percent is the rate. The sign for percent is '%'.

3.2 FRACTION TO RATE PERCENT

To convert (or express) any fraction $\frac{a}{b}$ to rate percent, multiply it by 100 and put a (%) sign.

$$\Rightarrow \boxed{\frac{a}{b}} = \boxed{\frac{a}{b} \times 100\%}$$

$\frac{a}{b}$: a fraction; $\frac{a}{b} \times 100\%$: rate percent

EXAMPLE: Express $\frac{3}{4}$ in rate percent

Required rate percent = $\frac{3}{4} \times 100\% = 75\%$

3.3 RATE PERCENT TO FRACTION

To convert a rate percent to a fraction, divide it by 100 and delete the % sign.

EXAMPLE: 8% can be converted to a fraction as $\frac{8}{100}$.

TABLE 3.1 Fractional Equivalents of Important Percentages

$1\% = \frac{1}{100}$	$2\% = \frac{1}{50}$	$4\% = \frac{1}{25}$	$8\% = \frac{2}{25}$	$16\% = \frac{4}{25}$	$64\% = \frac{16}{25}$	$96\% = \frac{24}{25}$
$5\% = \frac{1}{20}$	$10\% = \frac{1}{10}$	$20\% = \frac{1}{5}$	$40\% = \frac{2}{5}$	$60\% = \frac{3}{5}$	$80\% = \frac{4}{5}$	$120\% = \frac{6}{5}$
$6\frac{1}{4}\% = \frac{1}{16}$	$12\frac{1}{2}\% = \frac{1}{8}$	$25\% = \frac{1}{4}$	$37\frac{1}{2}\% = \frac{3}{8}$	$50\% = \frac{1}{2}$	$87\frac{1}{2}\% = \frac{7}{8}$	$100\% = 1$
$8\frac{1}{3}\% = \frac{1}{12}$	$16\frac{2}{3}\% = \frac{1}{6}$	$33\frac{1}{3}\% = \frac{1}{3}$	$66\frac{2}{3}\% = \frac{2}{3}$	$83\frac{1}{3}\% = \frac{5}{6}$	$133\frac{1}{3}\% = \frac{4}{3}$	

Note: Similarity along the horizontal rows are to be observed for memorizing table 5.1.

3.4 ORIGINAL NUMBER AND ITS RATE PERCENT

Rate percent of a number is the product of the rate percent and the number which gives a result.

$\Rightarrow p\%$ of number = result

$$\Rightarrow \frac{p}{100} \times \text{number} = \text{result}$$

$$\text{(Original) number} = \frac{\text{result}}{p} \times 100$$

$$\text{original number} = \frac{\text{result}}{\text{rate percent}} \times 100$$

EXAMPLE: 9% of what number is 36?

SOLUTION

Here the result is 36. Using relation

$$\text{Original number} = \frac{36}{9} \times 100 = 400$$

EXAMPLE: If 30% of a number is 48, then what is 70% of the number ?

SOLUTION

Here one result is known, but second result is to found

$$\frac{30 \times \text{number}}{70 \times \text{number}} = \frac{48}{x} \Rightarrow x = 112$$

(Here the definition of rate percent is used in the numerator and the denominator.)

3.4.1 Two Different Rate Percents of a Number

Two different rate percents of the same number will definitely give two different results.

Let the rate percents be p_1, p_2 and results be x_1, x_2

then p_1 % of number $N = x_1$

p_2% of same number $N = x_2$

Adding, $(p_1 + p_2)$% of number $= x_1 + x_2$

Subtracting, $(p_1 - p_2)$% of number $= x_1 - x_2$

EXAMPLE: If 40% of the number exceeds the 25% of it by 54, find the number.

SOLUTION

Using,

$(p_1 - p_2)$% of number $= x_1 - x_2$

$(40 - 25)$ % of number $= 54$

$$\text{Number} = \frac{54}{15\%} = \frac{54}{15} \times 100 = 360$$

3.4.2 Expressing a given quantity as a percentage of Another given quantity

Let one given quantity be x and another given quantity be y. It is often asked to find what percentage of y is x. Here both quantities (x and y) should be in same units. If not, they should be converted into the same unit.

Concept

The question requires us to express one given quantity x as a percentage of another given quantity y.

Since y is the basis of comparison, so, y will be in the denominator. But x is to be converted as percentage of y, hence x will be in the numerator of the fraction. Now to convert the fraction to percentage, we will multiply it by 100. So, we get

$$\text{the required percentage} = \frac{x}{y} \times 100\%.$$

EXAMPLE: To find 30 is what percent of 150 or 'what percentage of 150 is 30 ?

SOLUTION

Using the earlier concept, we find here that 150 is the basis of comparison and hence 150 will be in the denominator.

$$\text{The required percentage} = \frac{30}{150} \times 100\%$$

$$= 20\%.$$

EXAMPLE: What percentage of 320 is 86.4?

SOLUTION

Here, 320 is the basis of comparison and it will be in the denominator.

$$\therefore \quad \text{required percentage} = \frac{86.4}{320} \times 100\%$$

$$= 27\%$$

3.5 CONVERTING A PERCENTAGE INTO DECIMALS

Case I

Let the percentage be a positive integer, then place a decimal point after two places from the extreme right of the integer to convert it into a decimal. If the percentage is a single digit number, add one zero to the left of it and then place the decimal point for its conversion percentage sign is removed after conversion.

EXAMPLE: 67% may be converted into decimals as 0.67, because $67\% = \frac{67}{100} = 0.67$.

8% may be written as 0.08

↳ {Zero added to its left to make it a two digit number so that decimal point can be placed.}

253% is equivalent to 2.53.

Case II

Let the percentage be a decimal fraction.

The percentage being a decimal fraction, shift decimal by two places to the left. Add zero to the left of the fraction, if needed.

Two place left Shifted

EXAMPLE: 3.5% may be written as 0.035

0.7% may be written as 0.007

(Zero is added to its left so that decimal point can be shifted by two places to the left.)

Case III

Let the percentage be a fraction.

If the percentage is a fraction of the form $\frac{a}{b}$, then convert it into a decimal fraction and follow the rule detailed in case II.

EXAMPLE: $\frac{1}{4}$% is equivalent to 0.25% which may be converted into decimals as 0.0025.

3.6 CONVERTING A DECIMAL INTO A PERCENTAGE

In this case, the method of 5.6 is reversed, i.e., shift the decimal point two places to the right. Add zero to the extreme right if required. Then add % sign.

Shift

EXAMPLE: 0.45 may be expressed as 45%.

0.032 is equivalent to 3.2%

1.7 is equivalent to 170%

→ Zero is added so that decimal point can be shifted by two places.

3.7 EFFECT OF PERCENTAGE CHANGE ON ANY QUANTITY (NUMBER)

If any number (quantity) is increased by x%, then

$$\text{new number (quantity)} = \text{original number} \times \left(\frac{100+x}{100}\right)$$

or

= original number × (1 + decimal equivalent of x%).

Similarly, if any number (quantity) is decreased by x%, then

$$\text{new number (quantity)} = \text{original number} \times \left(\frac{100-x}{100}\right)$$

or

= original number × (1 − decimal equivalent of x%).

Note: In case of percentage decrease, a (−)ve sign is put before x, otherwise the formula is same.

EXAMPLE: The present salary of Q is ₹3000. This will be increased by 15% in the next year. What will be the increased salary of Q?

SOLUTION

Here, the salary is to be increased by 15%.

15% is equivalent to 0.15

∴ the increased salary = $3000\,(1 + 0.15)$ or $3000 \times \frac{100+15}{100}$

$= 3000 \times 1.15$

$= ₹\ 3450.$

3.8 TWO STEP CHANGE OF PERCENTAGE FOR A NUMBER

In the first step, a number is changed (increased or decreased) by x%, and in the second step, this changed number is again changed (increased or decreased) by y%, then net percentage change on the original number can be conveniently found out by using the following formula,

$$\text{net \% change} = x + y + \frac{xy}{100} \qquad (+ \text{ or } -) \qquad (2)$$

If x or y indicates decrease in percentage, then put a (−)ve sign before x or y, otherwise positive sign remains.

EXAMPLE: If a number is increased by 12% and then decreased by 18%, so find the net percentage change in the number.

SOLUTION

Using relation 2

$$\text{net \% change} = x + y + \frac{xy}{100}$$

where $x = 12 \qquad y = -18$

$$\Rightarrow \quad \text{net \% change} = 12 - 18 + \frac{(12)\times(-18)}{100}$$

$= -6 - 2.16$

$= -8.16$

(−) sign signifies that there is percentage decrease in the result. Therefore, −8.16 indicates net 8.16% decrease of the given number as a result of 12% increase and 18% decrease. It also implies that 12% increase and 18% decrease are equivalent to 8.16% decrease.

EXAMPLE: If the length of rectangle increases by 30% and the breadth decreases by 12%, then find the % change in the area of the rectangle.

SOLUTION

Since, length × breadth = area, and both the length and breadth are changed, so, using the formula (2A), we get

$$\text{net \% change in product} = x + y + \frac{xy}{100}$$

where $x = 30,\ y = -12$

$$\Rightarrow \text{net \% change in area} = 30 - 12 + \frac{30 \times -12}{100}$$
$$= 18 - 3.6$$
$$= +14.4$$

It implies that there is 14.4% increase in the area of the rectangle.

3.9 FIXED EXPENDITURE OF AN ITEM

Since, expenditure = consumption × rate of item, so for keeping the expenditure fixed it is essential that:
– If the rate increases, then consumption is to be reduced.
– If the rate falls, then consumption is to be increased.
Accordingly,

$$\text{\% change in consumption} = \frac{\text{\% change in rate}}{100 + \text{\% change in rate}} \times 100$$

If rate falls then % change in rate is (–) ve.
Above formula can also be used to find out.
(a)% change in either length or breadth, when area of a rectangle is to be kept fixed, [because area = length × breadth like expenditure = consumption × rate].
(b)% change in either time or speed, when distance covered is to be fixed [because distance = speed × time].

EXAMPLE: If the price of tea is increased by 10%, then by how much percentage must a house wife reduce her consumption, to have no extra expenditure?

SOLUTION

Since price × consumption = expenditure and expenditure has to be kept fixed (or unchanged), so, when the price increases by 10%,

$$\text{the \% reduction in consumption} = \frac{10}{100+10} \times 100\%$$
$$= 9\frac{1}{11}\%.$$

3.10 % EXCESS OR % SHORTNESS

When a number A exceeds the another number B by x %, then % shortness of $B = \frac{x}{100+x} \times 100$

It implies that B is less than A by $\frac{x}{100+x} \times 100\%$

Similarly, if a number A is short of (or less than) B by x%, then % excess of $B = \frac{x}{100-x} \times 100$

i.e., B is more than A by $\frac{x}{100-x} \times 100\%$

EXAMPLE: If the income of Amar is more than that of Samar by 25%, then by how much percentage Samar's income is less than that of Amar?

SOLUTION

Required % shortness (less) income of Samar

$$= \frac{25}{100+25} \times 100\%$$
$$= 20\%.$$

Therefore, income of Samar is 20% less than that of Amar

CHAPTER 4

Average

4.1 INTRODUCTION

The idea of **average** is not new to us. We all are familiar with the following types of statements:

(i) The average runs scored by Sachin Tendulkar in a series is 72.

(ii) The average marks secured by Kana is 78%.

If a man earns Rs 40, Rs 50, Rs 56, Rs 46, and Rs 48 on five consecutive days of a week, then he earns a total of Rs $(40 + 50 + 56 + 46 + 48) = \text{Rs } 240$.

To find his average earning per day, his total earning is divided by the number of days, i.e.,

$$\text{Average} = \frac{240}{5} = \text{Rs } 48$$

Average earning does not mean that he earned Rs 48 everyday. But had he earned Rs 48 everyday, then his total earnings would have also been Rs 240 in 5 days.

Hence, to find the average of given quantities:

Step 1 The given quantities are added to get a **sum**- after let

Step 2 The **sum** is divided by the **number of items** to get the **Average.**

$$\therefore \quad \frac{\text{sum of all the items}}{\text{number of items}} = \text{Average} \qquad \textbf{1}$$

Note: The average is also called the **mean.**

The quantities, whose average is to be determined, *should be in the same unit.*

Hence,

$$\text{Sum of all the items} = \text{Average} \times \text{no. of items} \qquad \textbf{2}$$

4.2 AVERAGE OF DIFFERENT GROUPS

Sometimes, the average of two different groups are known and the average of a third group (made by combining these two groups) is to be found out.

Let,

	Group 1	+	Group 2	makes	Combined Group (1 + 2)
No. of items =	m		n		$m + n$
Average =	a		b		A
Sum of all items =	ma		nb		$ma + nb$

Therefore, average of combined group

$$= \frac{\text{Sum of all items}}{\text{No of items}}$$

$$A^* = \frac{ma + nb}{m + n} \qquad \textbf{3}$$

*This formula is also applicable for more than two groups forming the combined group.

EXAMPLE The average weight of 17 girls is 20 kg and that of 23 boys is 22 kg. Find the average weight of the class.

* Here, it deals with ONE item only, so, average has got no meaning and thus, 'average' is replaced by 'value' of that ONE item.

SOLUTION

	Girls	Boys
No. in the class =	17	23
Average =	20	22

$$\therefore \text{ average weight of the class} = \frac{17 \times 20 + 23 \times 22}{17 + 23}$$

$$= 21.15 \text{ kg}$$

4.3 ADDITION OR REMOVAL OF ITEMS AND CHANGE IN AVERAGE

We know, $\text{Average} = \dfrac{\text{Sum of the items}}{\text{Number of items}}$

So, the original average may change (increase/decrease), if number of items change. The number of items may change in the following two cases,

Case I

When one new items is added

Let the average of N items = A

One new items is **added** and the average increases or decreases by x, then

Value* of the new item added = $\boxed{A \pm (N+1)x}$ 4

Case II

When one items is removed

Let the average of N items = A

One item is removed average increases or decreases by x.

Value* of the item removed = $\boxed{A \pm (1-N)x}$ 5

SOLVED EXAMPLES

CHAPTER 5

Ratio and Proportion

5.1 INTRODUCTION

(a) **Ratio** A ratio is a comparison of two numbers (quantities) by division. The ratio of a to b is written as,

$$a : b = \frac{a}{b} = a \div b$$

In the ratio $a : b$, a and b are called the terms of the ratio; a is the antecedent b is the consequent.

A ratio is a number, so to find out the ratio of two quantities, they must be expressed in the same units.

(b) **Proportion** A proportion is an expression which states that two ratios are equal.

e.g., $\frac{3}{12} = \frac{1}{4}$ is a proportion. It can also be expressed as $3 : 12 = 1 : 4$ or $3 : 12 :: 1 : 4$.

Each quantity in proportion is called **term** or **proportional**. The first and the last terms are called the **extremes,** whereas the second and the third terms are called the **middle terms**. When four quantities are in proportion, the last quantity is said to be **fourth proportional** to the other three and also we find **product of middle terms = product of extremes**

(Thus, if $a : b = x : y$, then $bx = ay$).

2nd term × 3rd term = 1st term × 4th term

e.g., in $4 : 8 = 12 : 24$, we have

$8 \times 12 = 4 \times 24$

5.2 PROPERTIES OF RATIO

(a) In a ratio, **two** quantities are **compared**. So, the quantities must be of the same kind, i.e., they must be expressed in the same units.

(b) The ratio of two quantities determines how many times one quantity is contained by the other.

(c) The order of the terms in a ratio $a : b$ is very important. Since $4 : 5$ is different from $5 : 4$.

5.3 DIVIDING A GIVEN NUMBER IN THE GIVEN RATIO

Let A be the given number. The given ratio is $a_1 : a_2$

Here A is to be divided in the ratio $a_1 : a_2$.

It implies that A is divided in two parts such that value of first part: value of second part $= a_1 : a_2$.

Therefore,

$$\text{first part} = \frac{a_1}{a_1 + a_2} \times A$$

$$= \text{first term of ratio} \times \left(\frac{\text{Sum of parts}}{\text{Sum of terms of ratio}} \right)$$

$$\text{second part} = \frac{a_2}{a_1 + a_2} \times A$$

$$= \text{Second term of ratio} \times \left(\frac{\text{Sum of parts}}{\text{Sum of terms of ratio}} \right)$$

Since A has been divided into two parts, so, first part + second part = A.

EXAMPLE: Two numbers are in the ratio 8:9. If the sum of the numbers is 119, find the numbers.

SOLUTION

Since the sum of two numbers is 119, so, the problem implies that 119 is divided in two parts in the ratio 8:9.

Therefore,

$$\text{first number} = \frac{8}{8+9} \times 119 = 56$$

$$\text{second number} = \frac{9}{8+9} \times 119 = 63 \text{ or } (119 - 56 = 63)$$

Note: These relations are also true for dividing a given number into more than two ratios (i.e., more than two parts).

When any number A is divided in more than one ratio such as $a : b : c : d : \ldots$

then,

$$\text{value of any part} = \frac{\text{its related ratio term}}{a + b + c + \ldots} \times A$$

e.g., third part $= \frac{c}{a+b+c+\ldots} \times A.$

EXAMPLE: Dividing Rs 3,200 among P, Q, R in the ratio 5:2:9, find the amount received by Q.

SOLUTION

Amount received by Q

$$= \frac{\text{its related ratio term}}{\text{sum of ratio terms}} \times \text{Total amount}$$

$$= \frac{2}{5+2+9} \times 3200$$

$$= \text{Rs } 400$$

5.4 COMPARISON OF RATIOS

Let $a : b$ and $c : d$ be two ratios. Then

$a : b > c : d$ if $ad > bc$

i.e., $\frac{a}{b} > \frac{c}{d}$ if $ad > bc$

Similarly,

$a : b < c : d$ if $ad < bc$

and $a : b = c : d$ if $ad = bc$

In brief, when we want to compare two ratios, i.e. whether 1st ratio >, =, or < 2nd ratio, then, it is necessary only to check,

1st term of 1st ratio	×	2nd term of 2nd ratio	>, =, or <	1st term of 2nd ratio	×	2nd term of 1st ratio

EXAMPLE: Compare 5:7 and 2:3.

SOLUTION

Given ratios are

5 : 7 and 2 : 3

(7 × 2)

2nd multiplication

1st multiplication

(5 × 3)

Here, $5 \times 3 > 7 \times 2$

(1st multiplication > 2nd multiplication)

So, $5 : 7 > 2 : 3$ (*1st ratio > 2nd ratio*)

Note: This method is an easier one to use.

5.5 RELATION AMONG THE QUANTITIES MORE THAN TWO

Number of quantities are **three.**

Given $a : b = x : y$ (say) and $b : c = p : q$ (say),

then the given quantities a, b, and c can be related as,

$a : b = x : y$

$b : c = p : q$ [*Follow the arrow and multiply to get the each term in the final ratio*]

$\Rightarrow$ $a : b : c = xp : py : yq$

$a : c = xp : yq$

EXAMPLE: If incomes of Ram and Shyam are in the ratio of 3 : 5 and that of Shyam and Mohan are in the ratio of 7 : 4, then find the ratio of incomes of Ram, Shyam, and Mohan.

SOLUTION

Ram : Shyam = 3 : 5

Shyam : Mohan = 7 : 4

(*Follow the arrow & multiply*)

Ram : Shyam : Mohan = $3 \times 7 : 5 \times 7 : 5 \times 4 = 21 : 35 : 20$

5.6 PROPORTION WHEN TWO RATIOS ARE ADDED EQUALLY

Let two containers have equal volume:

These two containers A and B have milk and water in the ratio $a : b$ and $p : q$ respectively.

Hence the Amount of milk in A $= \frac{a}{a+b}$,

milk in B $= \frac{p}{p+q}$

Quantity of water in B $= \frac{b}{a+b}$, water in B $= \frac{q}{p+q}$

Now, the total contents of two containers are put in a third container c.

Quantity of milk in C = quantity of milk in (A + B)

$$= \left(\frac{a}{a+b} + \frac{p}{p+q}\right)$$

Quantity of water in C = quantity of water in (A + B)

$$= \left(\frac{b}{a+b} + \frac{q}{p+q}\right)$$

$$\text{Reqd. ratio in C} = \frac{\text{milk}}{\text{water}} = \frac{\left(\frac{a}{a+b} + \frac{p}{p+q}\right)}{\left(\frac{b}{a+b} + \frac{q}{p+q}\right)}$$

Following examples will explain the above relation:

Example: Two equal glasses are respectively $\frac{1}{3}$ and $\frac{1}{4}$ full of milk. They are then filled up with water and contents are mixed in a tumbler. find the ratio of milk and water in the tumbler.

SOLUTION

First glass contain milk = $\frac{1}{3}$, water = $\frac{2}{3}$

Second glass contain milk = $\frac{1}{4}$, water = $\frac{3}{4}$

Quantity of milk in tumbler = $\frac{1}{3}+\frac{3}{4}=\frac{7}{12}$

Quantity of water in tumbler = $\frac{2}{3}+\frac{3}{4}=\frac{17}{12}$

Ratio of milk : water in tumbler = $\frac{7}{12}:\frac{17}{12}=7:17$

CHAPTER

6 Problems on Simple Equations

6.1 INTRODUCTION

A statement that two mathematical quantities are equal is called an equation.

An equation of the type $ax + b = 0$, $a \neq 0$ is called a linear equation in the variable x.

6.2 FRAMING AND SOLVING WORD PROBLEMS

Step 1 : Assume the unknown quantity to be x.

Step 2 : Frame a linear equation by using the condition given in the problem.

Step 3 : Solve and find the value of x.

In some cases, there may be two unknowns then we need to assume the smaller quantity to be x.

CHAPTER

7

Chain Rule

7.1 DEFINITION

When quantities of different kinds are connected to one another so that we know how much of one quantity is equivalent to a given quantity of a second, etc., we can determine how much of the last kind is equivalent to a given quantity of first kind by the *chain rule*.

7.2 DIRECT PROPORTION AND INDIRECT PROPORTION

If increase or decrease of a quantity Q_1 causes increase or decrease of another quantity Q_2 in the same extent, then, Q_1 is directly proportional to $Q_2 \Rightarrow Q_1 \propto Q_2$

I. Number of persons $\propto$ Amount of work done, i.e., more persons, more work

II. Number of days $\propto$ Amount of work, i.e., more days, more work

III. Working rate $\propto$ Amount of work, i.e., more working rate, more work

IV. Efficiency of man $\propto$ Amount of work, i.e., more efficiency of man, more work

Combining I, II, III, and IV, (Man × days × Workrate × Efficiency) $\propto$ Amount of work. If increase of a quantity Q_1 causes decrease of a quantity Q_2, in the same extent, then, Q_1 is indirectly proportional to $Q_2 \Rightarrow Q_1 \propto \dfrac{1}{Q_2}$

V. Number of men $\propto \dfrac{1}{\text{No.of days}}$, i.e., more the men, less the no. of days required.

7.2.1 Important Formula

(a) $$\frac{\text{Man}_1 \times \text{Days}_1 \times \text{Work rate}_1}{\text{Amount of work done}_1} = \frac{\text{Man}_2 \times \text{Days}_2 \times \text{Work rate}_2}{\text{Amount of work done}_2}$$

Remember, *"Man days" required per unit work is always same*. In fact, Man × Days specify the volume of job or work.

(b) If in place of men there are engines burning coal for certain number of hours, then the above equation changes to

$$\frac{\text{No. of Engine}_1 \times \text{Hours}_1 \times \text{Consumption Rate}_1}{\text{Amount of coal burnt}_1} = \frac{\text{Engine}_2 \times \text{Hours}_2 \times \text{Consumption Rate}_2}{\text{Amount of coal burnt}_2}$$

because, here the job of an engine is to burn the coal.

(c) If number of examiners examining a number of answer books in a number of days by working a number of hours or day, since the job of examiner is to check the answer books, then,

$$\frac{\text{No. of examiner}_1 \times \text{Days}_1 \times \text{Work rate}_1}{\text{No. of answer books checked}_1} = \frac{\text{Examiner}_2 \times \text{Days}_2 \times \text{Work rate}_2}{\text{No. of answer books checked}_2}$$

$\therefore$ From, (a), (b), and (c), it is concluded briefly, that,

$$\boxed{\frac{N_1 \times D_1 \times R_1 \times E_1}{W_1} = \frac{N_2 \times D_2 \times R_2 \times E_2}{W_2} \quad (1)}$$

where, N_1, N_2 = number of workers;
D_1, D_2 = time of work
R_1, R_2 = Work rate of worker or machine;
E_1, E_2* = efficiency of worker or machine
W_1, W_2 = Amount of work (of same nature) done

* Efficiency of worker or machine is taken to be unity (=1) if other wise, not specified in the problem.

† This formula can be derived by combining I, II, III, IV and V from 7.2.

SOLVED EXAMPLES

E-1 If 120 men can do a job in 100 days, in how many days will 150 men do it?

S-1 See, 7.2.1, $W_1 = W_2$

Since same amount of job is done.

$\therefore \quad N_1 \times D_1 = N_2 \times D_2$

$\Rightarrow \quad 120 \times 100 = 150 \times N_2$

$\Rightarrow \quad N_2 = 80$ days.

E-2 If 18 men working 5 hours a day for 8 days can complete a job, how many men working 8 hours a day for 6 days will be needed?

S-2 Using, $\frac{N_1 \times D_1 \times R_1}{W} = \frac{N_2 \times D_2 \times R_2}{W}$,

where $N_1 = 18,\ D_1 = 8,\ R_1 = 5$

$N_2 = ?,\ D_2 = 6,\ R_2 = 8$

$\Rightarrow 18 \times 8 \times 5 = N_2 \times 6 \times 8 \quad \Rightarrow \quad N_2 = 15$ men.

E-3 One thousand men in a fortress have provisions for 12 days. How long will the provisions last if 200 more men join them?

S-3 Here amount of work, i.e., amount of provisions is same (Since $N_1D_1 = N_2D_2$)

$\therefore \quad 1000 \times 12 = (1000 + 200) \times D_2$

$\Rightarrow \quad D_2 =$ **10 days.**

E-4 If 4 men reap 40 acres in 30 days, how many acres will 18 men reap in 12 days?

S-4 $\frac{N_1 \times D_1}{W_1} = \frac{N_2 \times D_2}{W_2}$ where, $N_1=4\ D_1=30,\ W_1=40$

$N_2=18\ D_2 = 12 \quad W_2 = ?$

$\Rightarrow \quad \frac{4 \times 30}{40} = \frac{18 \times 12}{W_2} \quad \Rightarrow \quad W_2 = 72$ acres.

E-5 If 12 men can build a wall 100 m long, 3 m high and 0.5 m thick in 25 days, in how many days will 20 men build a wall 60 m × 4 m × 0.25 m.

S-5 Here the amount of job is the volume of the wall built.

Since $\frac{N_1 D_1}{W_1} = \frac{N_2 D_2}{W_2}$

[W_1, W_2 are the volume of wall]

$\Rightarrow \frac{12 \times 25}{100 \times 3 \times 0.5} = \frac{20 \times D_2}{60 \times 4 \times 0.25} \Rightarrow D_2 = 6$ days.

E-6 Fifteen men take 21 days of 8 hours each to do a work. How many days of 6 hours each would 21 women take, if 3 women do as much work as 2 men?

S-6 Here, nature of worker is different. In one case it is man and in other case it is woman. So, woman needs to be converted to man.

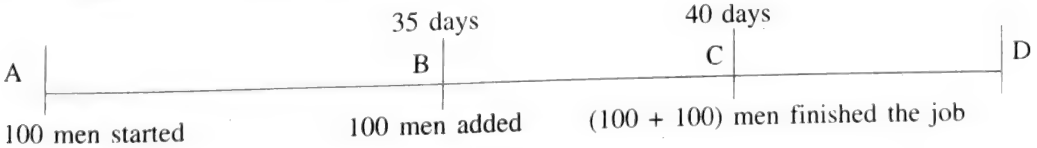
Since 3 women = 2 men. $\Rightarrow$ 21 women = 14 man.

$\therefore \quad 15 \times 21 \times 8 = 14 \times D_2 \times 6$

(Since $N_1 \times D_1 \times R_1 = N_2 \times D_2 \times R_2$)

$\Rightarrow \quad D_2 = 30$ days.

E-7 A contractor undertakes to do a piece of work in 40 days. He engages 100 men and after 35 days, he engaged an additional 100 men and completes the work. How many days behind the schedule would the work have been, if he had not engaged the additional men?



S-7 Change in number of men takes place at point B.

$\therefore$ portion after point B is to be considered.

$100 \times D_1 = (100 + 100) \times 5 \Rightarrow D_1 = 10$ days $= BD$.

$\therefore$ If 100 additional men were not engaged, $10 - 5 = 5$ days would the work have been behind the schedule. In diagram, CD represents the time behind the schedule.

E-8 A contract is to be completed in 56 days and 104 men were set to work, each working 8 hours a day. After 30 days, $\frac{2}{5}$ of the work is finished. How many additional men may be employed so that work may be completed on time, each man now working 9 hours per day?

S-8 $\frac{N_1 \times D_1 \times R_1}{W_1} = \frac{N_2 \times D_2 \times R_2}{W_2}$ → Remaining days

$\frac{104 \times 30 \times 8}{\frac{2}{5}} = \frac{(104 + x) \times (56 - 30) \times 9}{\left(1 - \frac{2}{5}\right)}$

→ Rest work to be done

$$\Rightarrow \quad \frac{104 \times 30 \times 8}{\frac{2}{5}} = \frac{(104+x) \times 26 \times 9}{\frac{3}{5}}$$

$x = 56$ men.

Hence, 56 additional men should be employed.

E-9 Ten men begin to work together on a job, but after some days, 4 of them leave. As a result, the job which could have been completed in 40 days is completed in 50 days. How many days after the commencement of the work did the 4 men leave?

S-9 Total job assigned = $10 \times 40 = 400$ men days.

Say, after x days of the commencement of work, 4 men leave.

$\therefore$ 4 men work for x days.

$\therefore \quad M_1D_1 = M_2D_2 + M_3 \times D_3$.

$10 \times 40 = 4 \times x + 6 \times 50 \quad \Rightarrow \quad x = 25$.

(Since 6 men work throughout 50 days)

They left 25 days after the commencement of work.

E-10 Four lorries carrying 4 tons each move 128 tons in 8 days. In how many days will 6 lorries carrying 3 tons each move 540 tons?

S-10 $\frac{N_1 \times D_1 \times R_1}{W_1} = \frac{N_2 \times D_2 \times R_2}{W_2}$

$$\frac{4 \times 8 \times 4}{128} = \frac{6 \times D_2 \times 3}{540} \quad \Rightarrow \quad D_2 = 30 \text{ days.}$$

E-11 A hostel has provisions for 250 students for 35 days. After 5 days, a fresh batch of 25 students were admitted to the hostel. Again after 10 days, a batch of 25 students left the hostel. How long will the remaining provisions survive?

S-11

After 5 days,	$D_1 = 10$ after 10 days,	$D_2 = x$ remaining x days
25 students added,	B 25 students left	C D

$N_1 = 250 + 25 = 275$ $\qquad$ $N_2 = 275 - 25 = 250$

Remaining food provisions at B (after 5 days) = $ND = 250 \times (35 - 5)$ student days.

$\therefore \quad ND = N_1D_1 + N_2D_2 \quad \Rightarrow \quad (250 \times 30) = 275 \times 10 + 250 \times x$

$\Rightarrow x = 19$. $\quad \therefore$ The remaining provisions would last for 19 days.

E-12 A garrison of 3000 men has provisions for 25 days, when given at the rate of 900 g per head. At the end of 11 days, a reinforcement arrives and it was found that now the provision will last 10 days more, when given at the rate of 840 g per head. What is the strength of reinforcement?

S-12 Let strength of reinforcement be x

Remaining food provisions after 11 days = $3000 \times (25 - 11) \times 900$

Total men after 11 days = $(3000 + x)$

$\therefore \quad 3000 \times 14 \times 900 = (3000 + x) \times 10 \times 840$

$\Rightarrow \quad x = 1{,}500$.

$\therefore$ The reinforcement had 1,500 men.

CHAPTER 8

Time and Work

8.1 BASIC CONCEPTS

In solving the problems on time and work, the following concepts are useful.

Concept 1

Total amount of a complete job (or assigned job) = always unless otherwise specified.

Concept 2

If A can finish a work in 't' days, then

1 day's work by $A = \frac{1}{t}$ part.

Concept 3

If 1 day's work by A is $\frac{1}{t}$ part, then

time taken by A to finish the work = t days.

Concept 4

If A can do a work in t_1 days and B can do the same work in t_2 days, then

$\rightarrow A$'s 1 day work + B's 1 day work = $\left(\frac{1}{t_1}+\frac{1}{t_2}\right)$ part

$\rightarrow (A+B)$' s 1 day work = A's 1 day work + B's 1 day work.

Concept 5

If A can do a work in t_1 days and B can finish the same work in t_2 days, then A and B to getter can finish the same work in $\frac{t_1 t_2}{t_1+t_2}$ days.

Concept 6

If A and B together can finish a work in T days and A alone can finish the same work in t_1 days, then

B alone can finish the same work in $\frac{T t_1}{t_1 - T}$ days.

Concept 7

If a fill pipe fills a tank in 't' hours, then, part of tank filled in 1 hour = $\frac{1}{t}$

$\Rightarrow$ 1 hour filling work by pipe = $\frac{1}{t}$

Concept 8

If an empty pipe can empty a tank in 'e' hours then, part of tank emptied in 1 hour = $-\frac{1}{e}$

Concept 9

If fill pipe A and fill pipe B can fill a tank in t_1 hrs and t_2 hrs respectively, then both pipes together can fill the same tank in $\frac{t_1 t_2}{t_1+t_2}$ hrs.

Concept 10

If fill pipes A and B can fill a tank in t_1 hrs and t_2 hrs respectively, but an empty pipe C can empty the fill tank in t_3 hours, then

1 hr work of A, B and C = $\frac{1}{t_1}+\frac{1}{t_2}-\frac{1}{t_3}$

If work is (+) ve, then tank is filled.

If work is (–) ve, then tank is emptied.

CHAPTER 9

Profit, Loss and Discount

9.1 INTRODUCTION

The following basic terms are very useful to solve the problems on profit and loss.

(a) **Cost price (CP):** It is the price at which the article is purchased.

(b) **Selling price (SP)**: It is the price at which the article is sold.

If the selling price is greater than the cost price, then there is a gain in the transaction.

However, if the selling price of an article is less than the cost price, then there is a loss in the transaction.

9.2 GAIN OR LOSS

$$\text{Gain} = \text{SP} - \text{CP}$$

$$\text{Loss} = \text{CP} - \text{SP}$$

Hence, the difference of selling price and cost price gives the amount of gain or loss.

9.3 GAIN % OR LOSS %

$$\text{Gain \%} = \left(\frac{\text{Gain}}{\text{Cost price}} \times 100\right)\%$$

$$\text{Loss \%} = \left(\frac{\text{Loss}}{\text{Cost price}} \times 100\right)\%$$

The gain % or loss % is calculated on the cost price.

9.4 FINDING SELLING PRICE FROM GAIN % OR LOSS %

Sometimes, the gain % or loss % are also known for an article with known CP, and it is required to find the selling price (SP).

$$\therefore \quad \text{SP (Selling price)} = \frac{100 + \text{gain \%}}{100} \times \text{CP}$$

In case of loss %,

$$\text{SP} = \frac{100 - \text{loss \%}}{100} \times \text{CP}.$$

9.5 FINDING COST PRICE FROM GAIN % OR LOSS %

When the gain % or loss % and the selling price (SP) are known, then the cost price can be calculated with the help of the following formula;

$$\text{CP} = \left(\frac{100}{100 + \text{gain \%}}\right) \times \text{SP}.$$

When there is loss %,

$$\text{CP} = \left(\frac{100}{100 - \text{loss \%}}\right) \times \text{SP}.$$

9.6 AN ARTICLE SOLD AT TWO DIFFERENT SELLING PRICES

If the same article is sold at two different selling prices, then, the gain % or loss % will also be different. In such case, we can find out the unknown by using the following direct formula;

$$\frac{\text{Sale price}_1}{100 + \text{gain}_1\ \%} = \frac{\text{Sale price}_2}{100 + \text{gain}_2\ \%}$$

In case of loss,

$$\frac{\text{Sale price}_1}{100 - \text{loss}_1\ \%} = \frac{\text{Sale price}_2}{100 - \text{loss}_2\ \%}$$

9.7 TWO ARTICLES SOLD AT THE SAME PRICE

When the two articles are sold at the same price and gain % on first article = loss % on second article, then there is always an overall loss in the transaction.

$$\text{Overall loss} = -\left(\frac{x}{10}\right)^2\ \%$$

x is gain % or loss %.

9.8 MARKED PRICE AND DISCOUNT

Let the cost price = CP, then the merchant marks the article at a price which is higher than CP.
This is known as marked price (MP). Now, merchant allows a discount of d % on marked price while selling it to the customer.
Hence, selling price, (SP) = customer price.

$$SP = MP - \text{discount}$$

If, the merchant by selling the article gains x% on it, then

$$SP = CP\,(1 + x\,\%) = MP - \text{discount}$$

$$SP = CP\,(1 + x\,\%) = MP\,(1 - d\,\%).$$

9.9 FALSE SCALE

A shopkeeper uses false scale (or false weight) to sell his goods to the customer. The value of false scale, here, is lower than the true scale (or true weight). Hence, he gains. Again the shopkeeper sells his goods at x % profit or loss. Combining these two—false scale effect and x % profit or loss, he makes an overall gain of G %; then

$$\frac{100+G}{100+x} = \frac{\text{true scale (or weight)}}{\text{false scale (or weight)}}$$

G – overall gain %
X - loss or gain %
If he sells at cost price, then $x = 0$.

$$\frac{100+G}{100} = \frac{\text{true scale}}{\text{false scale}}$$

CHAPTER 10

Time and Distance

10.1 DEFINITION

The speed of a moving body is the distance travelled by it in unit time. Hence,

$$\text{Speed } V = \frac{\text{Total distance travelled}\,(d)}{\text{Total time taken}\,(t)}$$

or Total distance travelled = speed × total time

$$d = V \times t$$

10.2 AVERAGE SPEED

If a moving body travels $d_1, d_2, d_3, \ldots d_n$ metres, moving with different speeds $V_1, V_2, V_3, \ldots V_n$ metres/seconds in time $t_1, t_2, t_3, \ldots t_n$ seconds respectively, then it is necessary to calculate the average speed of the body throughout the journey. If the average speed is denoted by V_a, then,

$$V_a = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Average speed for whole journey

If known values are $\begin{array}{|c|c|c|c|}\hline d_1 & d_2 & \ldots & d_n \\ \hline t_1 & t_2 & \ldots & t_n \\ \hline \end{array}$ then,

$$V_a = \frac{d_1 + d_2 + \ldots + d_n}{t_1 + t_2 + \ldots + t_n} \quad 1$$

If known values are $\begin{array}{|c|c|c|c|}\hline & & & \\ \hline V_1, t_1 & V_2, t_2 & \ldots & V_n, t_n \\ \hline \end{array}$ then,

$$V_a = \frac{V_1 t_1 + V_2 t_2 + \ldots + V_n t_n}{t_1 + t_2 + \ldots + t_n} \quad 2$$

since $d = vt$

If known values, are $\begin{array}{|c|c|c|c|}\hline d_1 & d_2 & \ldots & d_n \\ \hline V_1 & V_2 & \ldots & V_n \\ \hline \end{array}$ then,

$$V_a = \frac{d_1 + d_2 + \ldots + d_n}{\frac{d_1}{V_1} + \frac{d_2}{V_2} + \ldots + \frac{d_n}{V_n}} \quad 3$$

since $t = \dfrac{d}{V}$

EXAMPLE A car travels 600 km in 11 hours and another 800 km in 17 hours. Find the average speed of the car during the entire journey.

SOLUTION

Here, distance and time are known.

$$\therefore \quad \text{Average speed} = \frac{d_1 + d_2 + \ldots + d_n}{t_1 + t_2 + \ldots t_n}$$

$$= \frac{600 + 800}{11 + 17} = 50 \text{ km/hour}$$

$\therefore$ the average speed is 50 km per hour.

EXAMPLE A bus moves 300 km at a speed of 45 km per hour and then it increases its speed to 60 km per hour to travel another 500 km. Find the average speed of the bus.

SOLUTION

Here, distance and speed are known. Time is not known.

$\therefore$ we use the Relation 3

$$\text{average speed} = \frac{d_1 + d_2 + \ldots + d_n}{\frac{d_1}{V_1} + \frac{d^2}{V^2} + \ldots + \frac{d_n}{V_n}}$$

As the time is not known, so the denominator of the previous problem, i.e., time has been replaced by $\frac{\text{distance}}{\text{speed}}$

Hence, $t_1, t_2, \ldots t_n$ have been replaced by $\frac{d_1}{V_1}, \frac{d^2}{V^2} \ldots \frac{d_n}{V_n}$

$$\Rightarrow \quad \text{average speed} = \frac{300+500}{\frac{300}{45}+\frac{500}{60}}$$

$$= \frac{800}{\frac{20}{3}+\frac{25}{3}}$$

$$= \frac{800\times 3}{45}$$

$$= 53\frac{1}{3} \text{ km per hour.}$$

$\therefore$ the average speed is $53\frac{1}{3}$ km per hour.

10.3 DISTANCE COVERED IS SAME

Let a body moving with two different speeds, say V_1 and V_2, cover the same distance d, in time t_1 and t_2, respectively then,

$$d = V_1 t_1 \Rightarrow t_1 = \frac{d}{V_1} \Rightarrow V_1 = \frac{d}{t_1} \quad \text{(i)}$$

$$d = V_2 t_2 \Rightarrow t_2 = \frac{d}{V_2} \Rightarrow V_2 = \frac{d}{t_2} \quad \text{(ii)}$$

we can use from (i) and (ii)

$$\frac{d}{V_1} - \frac{d}{V_2} = t_1 - t_2 \qquad 4$$

or, $$\frac{d}{t_1} - \frac{d}{t_2} = V_1 - V_2 \qquad 5$$

EXAMPLE An increase in the speed of a car by 10 km per hour saves 30 minutes in a journey of 100 km. Find the initial speed of the car.

SOLUTION

Here, distance, difference in time and change (difference) in speed are known, but speed* is to be found out.

Let the initial speed of the car $= V$ km/h

The new speed of the car $= (V + 10)$ km/hr.

using $\frac{d}{V_1} - \frac{d}{V_2} = t_1 - t_2$

$$\Rightarrow \quad 100\left[\frac{1}{V} - \frac{1}{V+10}\right] = \frac{30}{60}$$

$$\Rightarrow \quad \frac{100\times 10}{V(V+10)} = \frac{1}{2}$$

$$\Rightarrow \quad V(V+10) = 40 \times 50$$

Without further solving, it appears that $V = 40$ km/h.

$\therefore$ the initial speed of the car is 40 km per hour.

using $\frac{d}{t_1} - \frac{d}{t_2} = V_1 - V_2$

$$\frac{600}{t} - \frac{600}{t+5/2} = 12$$

$$\Rightarrow \quad 600\,(5/2) = t\,(t+5/2)\times 12$$

$$\Rightarrow \quad t\left(t+\frac{5}{2}\right) = 125$$

$$\Rightarrow \quad t = 10$$

$\Rightarrow$ required time $= 10 + 2\frac{1}{2} = 12\frac{1}{2}$ hrs

10.4 DISTANCE COVERED IS DIFFERENT

Let the moving body cover the distances d_1 and d_2 at speeds V_1 and V_2 in time t_1 and t_2, respectively.

Using distance = speed × time, we can make the various arrangements as

① $d_1 = V_1 t_1$	② $d_2 = V_2 t_2$	→ **distance reference**
① $t_1 = \frac{d_1}{V_1}$	② $t_2 = \frac{d^2}{V_2}$	→ **time reference**
① $V_1 = \frac{d_1}{t_1}$ 6	② $V_2 = \frac{d^2}{t_2}$ 7	→ **speed reference**

Specific use of these three references has been explained in the subsequent examples.

EXAMPLE A car travels for 11 hours. Out of this, it travels 100 km at a certain speed and then it increases its speed by 15 km/hr to cover the remaining 280 km. Find the time it takes to travel the span of 280 km.

SOLUTION

Given that

Speed difference	total time	d_1	d_2
$V_1 - V_2$	$t_1 + t_2$	100	280
15	11		

Use $(V_1 - V_2)$, because, here V_1 and V_2 are *not* to be found out separately. (speed reference is used)

$$V_1 - V_2 = \frac{d_1}{t_1} - \frac{d^2}{t^2}$$

$$\Rightarrow \quad 15 = \frac{280}{x} - \frac{100}{11-x}$$

(assume, x = time to travel 280 km)

$$\Rightarrow \quad 3 = \frac{56}{x} - \frac{20}{11-x}$$

$$\Rightarrow \quad x = 7$$

$\therefore$ the car travels the span of 280 km in 7 hours

EXAMPLE A car travels a distance of 170 km in 2 hours partly at a speed of 100 km/hr and partly at 50 km hr. Find the distance travelled at speed of 100 km/hr.

SOLUTION

Given that:

total distance	total time	V_1	V_2
$(d_1 + d_2)$	$(t_1 + t_2)$	100	50
170	2		

Use $(t_1 + t_2)$, because, here, t_1 and t_2 are *not* to be found out separately.

$$\therefore \quad t_1 + t_2 = \frac{d_1}{V_1} + \frac{d_2}{V_2} \quad \text{(Time reference is used)}$$

$$\Rightarrow \quad 2 = \frac{x}{100} + \frac{170 - x}{50}$$

(assume x = distance covered at 100 km/hr)

$$\Rightarrow \quad x = 60$$

$\therefore$ The car travelled 60 km at the speed of 100 km/hr.

EXAMPLE If a truck travels a distance of 240 km in 6 hours, partly at a speed of 60 km/hr and partly at 30 km/hr, then find the time for which it travels at 60 km/hr.

SOLUTION

Given that

total distance	total time	V_1	V_2
$d_1 + d_2$	$t_1 + t_2$		
240	6	60	30

Use $(d_1 + d_2)$, because here d_1 and d_2 are not to be found out separately.

So, using distance reference, we find,

$$d_1 + d_2 = V_1 t_1 + V_2 t_2.$$

$$\Rightarrow \quad 240 = 60 \times x + 30 \times (6 - x),$$

(assume x = time of travel at 60 km/hr)

$$\Rightarrow \quad x = 2$$

$\therefore$ the truck travels 2 hours at 60 km/hr.

10.5 STOPPAGE TIME PER HOUR FOR A TRAIN

For the same distance of travel, if a train runs

at average speed V_1 km/hr, without stopping

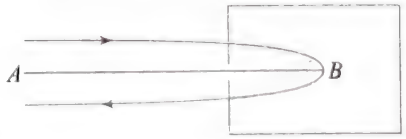
and at average speed V_2 km/hr, with stoppage, then

$$\text{stoppage time per hour} = \frac{V_1 - V_2}{V_1} \text{ hour.}$$

$$= \frac{\text{difference in speed}}{\text{faster speed}} \text{ hour} \qquad 8$$

10.6 TIME TAKEN WITH TWO DIFFERENCE MODES OF TRANSPORT

A man uses one or two different kinds of transport for going from A to B and back to A again.



He has three options available for the above journey:

Option

1. He uses fast transport both ways (i.e., A to B and back to A).
2. He uses slow transport both ways.
3. He uses mixed transport i.e., fast transport for one way and slow transport for the other way.

(Time taken by any one transport both ways – time taken by mixed transport) = time gained or lost*

[* in case of time loss, put a (–) ve sign before it.]

EXAMPLE A man takes 4 hours 30 minutes in walking to a certain place and riding back. He would have gained 1 hour 45 minutes by riding both ways. How long would he take to walk both ways?

SOLUTION

Using the Relation (9),

time for walking bothways – time by mixed (i.e., walking + riding) = time gained

$\Rightarrow$ time for walking both ways $-4\frac{1}{2} = +1\frac{3}{4}$

$\Rightarrow$ time for walking both ways $= 4\frac{1}{2} + 1\frac{3}{4}$

$= 6\frac{1}{4}$ hours.

$\therefore$ The man will take $6\frac{1}{4}$ hours to cover the same distance if he walks both ways.

10.7 TIME AND DISTANCE BETWEEN TWO MOVING BODIES

Let there be two persons, A and B.

Speed of $A = V_1$ km/hr

Speed of $B = V_2$ km/hr

If they walk in same direction,

A and B will be $(V_1 - V_2)$ km apart in 1 hour.

$\Rightarrow$ A and B will be 1 km apart in $\left(\frac{1}{V_1 - V_2}\right)$ hour

$\Rightarrow$ A and B will be x km apart in $\left(\frac{x}{V_1 - V_2}\right)$ hour

Similarly, if they walk in opposite directions, then A and B will be $(V_1 + V_2)$ km apart in 1 hour

$\Rightarrow$ A and B will be 1 km apart in $\left(\frac{1}{V_1 + V_2}\right)$ hour

$\Rightarrow$ A and B will be x km apart in $\left(\frac{x}{V_1 + V_2}\right)$ hour

EXAMPLE Two men, P and Q, start walking from a hotel at 2 km and $2\frac{1}{2}$ km an hour respectively. By how many km will they be apart at the end of $3\frac{1}{2}$ hours, if

(i) they walk in opposite directions

(ii) they walk in the same direction.

SOLUTION

(i) When they walk in opposite direction, then P and Q will be $\left(2 + 2\frac{1}{2}\right)$ km or $4\frac{1}{2}$ km apart in 1 hour.

$\therefore$ at the end of $3\frac{1}{2}$ hours, they will be $3\frac{1}{2} \times 4\frac{1}{2}$ km $= 15\frac{3}{4}$ km apart.

(ii) When they walk in the same direction, then P and Q will be $\left(2\frac{1}{2} - 2\right)$ km or $\frac{1}{2}$ km apart in 1 hour

$\therefore$ at the end of $3\frac{1}{2}$ hours, they will be $\frac{1}{2} \times 3\frac{1}{2}$ km $= 1\frac{3}{4}$ km, apart.

CHAPTER 1

Data Interpretation - Tables

1.1 INTRODUCTION

All sorts of figures, statistics, statements, etc. relating to any event (ranging from, contribution of different sectors to GDP growth of India, to the scheduling of a train) are termed as data. But data, as such, is presented in an organised form so that the desired information can be obtained easily from it.

This act of obtaining various meaningful informations, drawing important conclusions and also of making vital inferences from the given organised data is known as Data Interpretation.

Techniques for Interpretation of Data

Mostly used tools for interpretation of a data (or a group of data) are,

- Ratio
- Percentage
- Rate
- Average

Methods of Presenting Numerical Data

The numerical data pertaining to any event can be presented by any one or more of the following methods,

(a) Tables
(b) Line Graphs
(c) Bar Graphs
(d) Pie Charts (or Circle Graphs)
(e) Venn Diagram

Each of the above methods are being explained subsequently with examples.

1.2 RATIO

1.2.1 Techniques to Interpret the Data Involving Ratio

— In the simplest possible form, a ratio is a quotient or the numerical quantity obtained by dividing one figure by the other with same units.

— A ratio indicates what multiple, part or parts the **first quantity is of the second.**

For example, in a school, $\frac{\text{no. of girls}}{\text{no. of boys}} = K = \text{ratio}$

$\Rightarrow$ no. of girls = K times the no. of boys

or, no. of boys = $\frac{1}{K}$ of the no. of girls

— Ratio can be distinguished in terms of what is used as the base of comparison—that is the **denominator of the fraction.**

For example, in a school, girls or boys as proportion of total students,

$$\Rightarrow \quad \text{required ratio} = \frac{\text{no. of girls}}{\text{total no. of students}}$$

$$\text{or} \frac{\text{no. of boys}}{\text{total no. of students}} = \text{another ratio}$$

here, total no. of students is the basis of comparison.

Following techniques are useful to interpret the data involving ratio,

R(1) To evaluate a ratio, say, $\frac{3}{2684}$, where numerator is very small compared to the denominator, it is always better to reverse it and then divide **(Reverse operation).** as

$$\begin{array}{r} 3\overline{)2684}\,(894 \\ \underline{24} \\ 28 \\ \underline{27} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

$\therefore$ the given ratio $\approx \frac{1}{894}$

R(2) To evaluate a ratio, say $\frac{21.32}{796}$, where numerator << denominator and numerator is a decimal number, it is always convenient to approximate the fraction to a closest one involving integers only and then apply the reverse operation.

Therefore, $\frac{21.32}{796} \approx \frac{21}{796}$, then divide 796 by 21 results 38

$$\approx \frac{1}{38} \approx \frac{2}{76} \approx \frac{2}{74}$$

So, the ratio obtained $\left(\text{i.e., } \frac{14.37}{241}\right)$ during a data analysis may be close to the given choice (b)

(a) $\frac{3}{32}$ (b) $\frac{4}{67}$

(c) $\frac{7}{11}$ (d) $\frac{5}{81}$

R(3) To find the highest and the lowest among the ratios (< 1) when numerator << denominator.

Step 1 Do not calculate the result of each ratio.

Step 2 Try to do mental calculation for comparing the ratios as discussed below. As the comparison of the ratios have been asked for, so, calculating the value of ratio will unnecessarily consume some time.

For example, say, the ratios obtained in analysis of data are, $\frac{13}{447}, \frac{45}{508}, \frac{28}{583}, \frac{18}{531}$, to find the highest and the lowest among them, following method is adopted for **mental calculation.**

Apply reverse operation, i.e., straight way divide the denominator of the ratio by the numerator to find how many times the denominator is of the numerator.

$\Rightarrow \quad \frac{447}{13}, \frac{508}{45}, \frac{583}{28}, \frac{531}{18}$

No. of times >30, 10 20 <30 (closer value is not required for comparison)

↓ ↓

maximum value minimum value of no. of times

↓ ↓

minimum ratio maximum ratio

Since, reverse operation has been done, so, maximum no. of times will indicate the minimum ratio and minimum no. of times correspond to maximum ratio.

R(4) To find the highest and the lowest among the ratios (<1) when numerator < denominator.

Step 1 Approximate the given ratio (if the no. of digits in numerator/denominator is more than 2)

Step 2 Multiply the numerator by 10 and get the result fraction

Step 3 Find only the integer value of the result fraction

Step 4 If any of the integer value of the result fraction is same then find the next decimal place and so on.

Step 5 Compare the value of the result fractions. The maximum ratio will have maximum value.

For example, the ratios are $\frac{471}{525}, \frac{324}{411}, \frac{648}{749}$

Step 1 Approximated as $\frac{47}{52}, \frac{32}{41}, \frac{65}{75} \left(\approx \frac{13}{15}\right)$

Step 2 Multiply by 10, $\frac{470}{52}, \frac{320}{41}, \frac{130}{15}$

Step 3 Find the integer value, >9, <8, <9

Further calculation is not required because one fraction >9 and other <8.

Hence $\frac{471}{525}$ is the highest ratio and $\frac{324}{411}$ is the lowest ratio.

R(5) To find which of the given ratio is more or less than a specified limit.

Consider, $\frac{31}{83}, \frac{15}{46}, \frac{22}{65}$, then it is required to find which of the among ratio is less than $\frac{1}{3}$

It is convenient to check which denominator is $\frac{3}{1} = 3$ times the numerator. So, we find that

$$\frac{83}{31} < 3, \quad \boxed{\frac{46}{15} > 3,} \quad \frac{65}{22} < 3$$

↓

required ratio

Rule

If the denominator is more by 'K' times the numerator, then the ratio or given (fraction) is less by $\frac{1}{K}$

Similarly, if the denominator is less by K times the numerator, then the given ratio (fraction) is more by $\frac{1}{K}$.

1.3 PERCENTAGE

Ratios or proportions with the base 100 are known as **percentages** (%). For example, a ratio $\frac{x}{y}$ becomes $\frac{x}{y} \times 100$ in percentage form, and it denotes-what part of y is x if y is 100.

Percentage is a very useful tool for comparison in the analysis of data. For example, the ratio of girls to total students in school P and school Q are $\frac{1174}{2333}$, and $\frac{665}{1406}$.

This can however be better comprehended in a percentage form, which is for,

$$\text{School A} = \frac{1174}{2333} \times 100 = 50.3\ \%$$

$$\text{School B} = \frac{665}{1406} \times 100 = 47.3\ \%$$

This reveals that the girl ratio is little over 50% in school A, i.e., no. of girls is more than the no. of boys, on the other hand, school B with 47.3% girl ratio has a predominance of boys.

1.3.1 Techniques to Interpret Data Involving Percentages

P(1) *To find by how much per cent X is more or less than Y (or over Y) or compared to Y.*

$$\text{Required percentage} = \frac{\text{value of } X - \text{value of } Y}{\text{value of } Y} \times 100$$

(value of Y → base value for comparison)

So, the denominator part contains the value with which the comparison is made.

NB: than / over / compared to Y means the base of comparison is value of Y and hence, the value of Y will be in the denominator.

P(2) *To find the percentage change in any value in year M compared to that value in year K (or over year K)*

Compared to year K (or over year K) means the base of comparison is the value in year K.

$$\text{Required \% change } (x) = \frac{\text{value (year } M) - \text{value (year } K)}{\text{Value (Year } K)} \times 100$$

(value (year K) → base year; Value (Year K) → only base year in denominator)

if x is (+)ve, then there is % increase in the value in year M over year K (or percentage growth)

if x is (–)ve, then there is % decrease (or percentage decline).

P(3) *To find the original value if the value after its x% increase or decrease is known*

$$\text{Original value} = \frac{\text{value after } x\% \text{ increase or decrease}}{\left(1 \pm \frac{x}{100}\right)}$$

$+x$ for x% increase, (–) for x% decrease

EXAMPLE In 1998, the production of tea is 1584 million kg, which is 20% more than that in 1991. Find the production of tea in 1991.

⇒ Production of tea in 1991 = original value = base value for comparison and production of tea in 1991 has increased by 20% to reach 1584 million kg in 1998.

Hence, production of tea in 1991 $= \frac{1584}{1 + \frac{20}{100}} = \frac{1584}{1.20} = 1320$ million kgs

Similarly, if in 1998, the production of tea is 1584 million kg which is 20% less than that in 1991 find the production of tea in 1991.

Production of tea in 1991 $= \frac{1584}{1 - \frac{20}{100}} = \frac{1584}{0.80} = 1980$ million kgs

P(4) *Two percentage values cannot be compared unless the base values are known*

EXAMPLE 40% of male are literate in 1981 and 30% of the male are literate in 1991.

Obviously the base value (i.e., male population) in 1981 and in 1991 are different. Without knowing these base values it cannot be determined in which year (1981 or 1991) the no. of male literate is more.

P(5) *Two percentage values can be compared in terms of percentage values (but not in absolute values) when the base values are same (although base values are not known).*

EXAMPLE 40% of male are literate in 1981 and 30% of the male are literate in 1991. Male population in 1981 = Male population in 1991.

Now comparison of % values in 1981 and in 1991 reveals the following

— Male literates in 1981 is more than that in 1991 by.

$$\frac{40 - 30}{30} \times 100 = 33\frac{1}{3}\%$$

— Male literates in 1991 is less than that in 1981 by

$$\frac{30 - 40}{40} \times 100 = 25\%$$

P(6) *To find the value of a parameter in any year K when the % change in its value over two different periods (any period having the year K) are same.*

EXAMPLE

(in '000 tonnes)

Year	1994	1995	1996	1997	1998
Production	300	336	405	783	1008

Find the production of steel in 1999 if the % increase in (1998-1999) is same as the % increase in production during 1995-1996.

% change in production during = % change in production during

then, $\frac{336}{405}$ (1995-96) $= \frac{1008}{x}$ (1998-1999)

⇒ x = production of steel in year 1999 = 1215 thousand tonnes.

Important: Note the sequence of putting the values in the numerator and in the denominator.
On the concept of base year it may also be written as

$$\frac{\text{value (any year)}}{\text{value (related base year)}} = \frac{\text{value (any year)}}{\text{value (its related base year)}}$$

1.4 AVERAGE

$$\text{Average} = \frac{\text{sum of values of all the quantities}}{\text{number of quantities}}$$

The average is also called the **arithmetic mean**.
NB: All the quantities in the numerator must be in the same unit, otherwise average cannot be calculated.

1.4.1 Techniques to Interpret Data Involving Calculation of Averages

A(1) *To find the average of a parameter during given period of time. Let the parameter X assumes values $x_1,...x_n$ in n different years during given period of time,*
then annual average of parameter X

$$= \frac{\text{sum of the values}}{\text{no. of years}} = \frac{x_1 \ldots + x_n}{n}$$

Similarly, if the parameter X would have the values $x_1...x_n$ in n different months, then
monthly average of parameter X (or average value of parameter X during any period)

$$= \frac{\text{Sum of the values (monthwise) during that period}}{\text{no. of months}}$$

A(2) *To find the average % change in value of a parameter during a given period of time*

Average % change during the period

$$= \frac{\text{Percentage change during the given period}}{\text{No. of years (or months) or no. of entries}}$$

e.g.,

Figures in '000

Year	1994	1995	1996	1997	1998
No. of scooters	20	25	38	47	55

Find the average annual percentage increase in no. of scooters over the period 1994 to 1998?

% increase (or change) in no. of scooters during 1994-98

$$= \frac{55 - 20}{20} \times 100 = 175\%$$

Average % increase (or change) during 1994-98

$$= \frac{\text{\% increase during 1994-98}}{\text{No. of entires during 1994-98}}$$

$$= \frac{175}{5}\% = 35\%$$

A(3) *To find the value of which year (or the entry in a table) is closer to the average value of a given period*

Step 1 Find the average value for the given period

Step 2 Find the difference = any value (or entry) − average value. Minimum the difference, closer the value to average.

Step 3 If the **difference** is **same** for any two different entry (or values) then find the

$$\text{\% variation over the average} = \frac{\text{difference}}{\text{average}} \times 100$$

Since difference is same, so,
More the average, less the % variation, closer the value to the average.

EXAMPLE Production figures of five types of car by a company is given as below

Type of Cars

Year	A	B	C	D	E	Total
1995	20	10	15	8	20	73
1996	12	12	18	12	12	66
1997	24	14	17	14	15	84
1998	26	10	16	15	9	76
1999	22	16	14	16	11	79

EXAMPLE 1 In which of the given years the production of type D car was close to its average production over the years?

EXAMPLE 2 In which of the given years the production of type D car was close to the average production of all the cars in the year?

SOLUTION

The average production of type D car over the years $= \frac{8+12+14+15+16}{5} = \frac{65}{5} = 13.$

Difference with average is same (=1) for 1996 and 1997, and the average is same (=13). So, the production of type D car in both 1996 and 1997 are close to its average production over the years.

SOLUTION

It has striking difference (dissimilarity) from the above question because the value of average varies for each year (i.e. horizontally)

Average production of all cars in various years are

Year	Average production	Production of D type car	
1995	$= \frac{73}{5} \approx 14$	8	Difference with average = 14 – 8 = 6 is very wide, so calculation not required
1996	$= \frac{66}{5} \approx 13$	12	Difference with average = 14 – 8 = 6 is very wide, so calculation not required
1997	$\approx \frac{84}{5} \approx 17$	14	Difference with average = 14 – 8 = 6 is very wide, so calculation not required
1998	$\approx \frac{76}{5} \approx 15.2$	15	
1999	$= \frac{79}{5} \approx 15.8$	16	Calculate % variation, because difference with average is same(= 0.2)

Now, % variation $= \frac{\text{difference}}{\text{average}} \times 100$

Since difference is same in 1998 and 1999, so, the year with more average, will have less % variation and closer to average value.

In 1999, the value (or production) of *D* type car is closer to the average value of all the cars in that year.

A(4) *To find the average by mental calculation & choose the (closest) best alternative*

Sometimes the calculation of average would take a long time. In such cases it is better to do mental calculation and to check which of the given (or alternatives) comes close to the value.

NB: However, the actual calculation of average may be needed if more than one choices are closer to each other.

EXAMPLE: Production & Exports of Tea (in million kg) has been given below

Year	1990-91	1991-92	1992-93	1993-94	1994-95
Export	207	189	209	215	220
Production	421	561	587	645	660

The average proportion of tea exported as a fraction of the tea produced in the half decade was

(a) 0.11 (b) 0.22
(c) 0.36 (d) 0.41
(e) 0.49

SOLUTION

Here none of the given choices are closer to each other, so mental calculation may work. Just observe that exports are generally $\frac{1}{3}$rd of production with one year (1990-91) showing nearly $\frac{1}{2}$ of production as exports. Hence the required average will be $> \frac{1}{3}$ (= 0.33). Thus, 0.36 looks the closest amongst the alternatives given.

A(6) *To find the weighted average value of a parameter X.*

Let the parameter assumes different values as x_1, x_2, x_3 with weights w_1, w_2 and w_3 respectively, then

$$\text{Weighted average of } X = \frac{w_1x_1 + w_2x_2 + w_3x_3}{w_1 + w_2 + w_3}$$

The weights are also called frequencies. In the weighted average value of a parameter, it is seen that the each value of the variable and also the weights (frequency) of each value are considered.

If equal importance (or weight) is given to all the values, i.e. $w_1 = w_2 = w_3$, it reduces to simple average $= \frac{x_1 + x_2 + x_3}{3}$

EXAMPLE Sectoral Growth Rate of Major Industry Groups (1996-97) are given as

Sectors	Weights	1996-97
Mining	11.46	2.9
Manufacturing	77.11	8.7
Electricity	11.43	7.1
Overall	100.00	

Find the weighted average growth rate of all the industries in 1996-97 (or overall weighted industrial growth rate in 1996-97)

SOLUTION

Required weighted average growth rate in 1996-97

$$= \frac{11.46 \times 2.9 + 77.11 \times 8.7 + 11.43 \times 7.1}{11.46 + 77.11 + 11.43}$$

$$= 100 \text{ (as overall weight)} = \frac{785}{100} = 7.85$$

1.5 TABLES

It is the systematic and scientific presentation of quantitative data in a tabular form to understand the given information and to elucidate the problem in a certain field of study.

A table has the following six essential elements,

(i) **Title**—heading of the table

(ii) **Stule**—the section of the table containing row headings

(iii) **Column Captions**—the heading of each column

(iv) **Body**—consists the numerical figures

(v) **Footnotes**—further explanation of the table

(vi) **Source**—authority of the data

The interpretation of data of the tables is done on the basis of assessment of numerical figures in the light of inter-related column captions and style of the table. In fact, more attention is focussed on the relative position of different items (sub-entries) in a table and questions are answered mostly regarding:

(a) maximum and minimum value

(b) average value

(c) maximum and minimum ratio of any two parameters in column/row

(d) maximum/minimum rate of increase/decrease

(e) percentage

(f) items showing eccentric behaviour

SOLVED EXAMPLES

CHAPTER 2

Data Interpretation - Graphs

2.1 LINE GRAPH

A line graph (or cartesian graph) indicates the variation of a quantity with respect to the two parameters calibrated on the X and Y axis respectively.

The graph drawn indicates the production trend of a company X during the period 1990 to 1994. Graphs present a pictorial representation of the data. It is very useful in determining the trends and rate of change.

In most of the cases, X-axis contain the time parameter (may be year or month) and Y-axis represent any other variable parameter which have different values with respect to time.

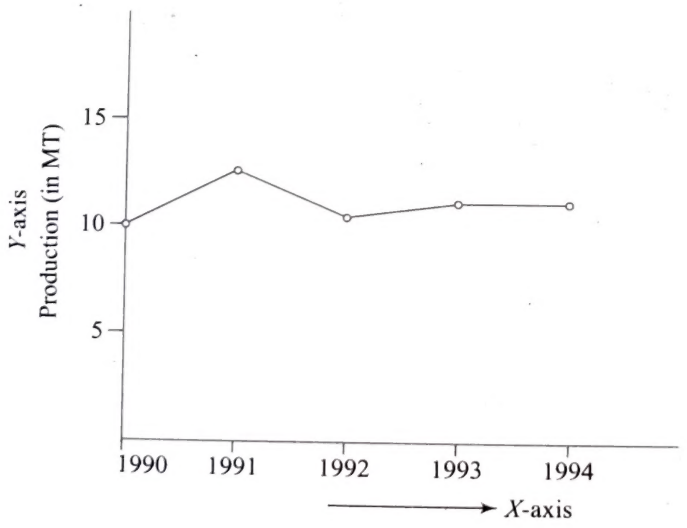


Fig. 2.1

In the line graph following points are to be remembered

(a) Any part (of the line graph) which is parallel to X-axis represents no change in the value of Y parameter with respect to value of X-parameter.

(b) The steepest (or maximum) part of the line graph indicates maximum percentage change of the value during the two consecutive periods in which the related part lies.

(c) If the steepest part is a *rise slope*, then it is *highest* percentage growth, and if the steepest part is a *decline slope*, it will represent a *maximum* percentage *fall* of the value calibrated in the other axis.

2.2 CHARTS

Charts are of two types—(i) Bar charts and (ii) Pie charts

2.2.1 Bar Charts

Bar charts are diagramatic representation of a discrete data. Various types of bar charts are

(a) Simple Bar Diagrams A simple bar diagram relates to *only one* variable. The values of the variable may relate to different years or different items.

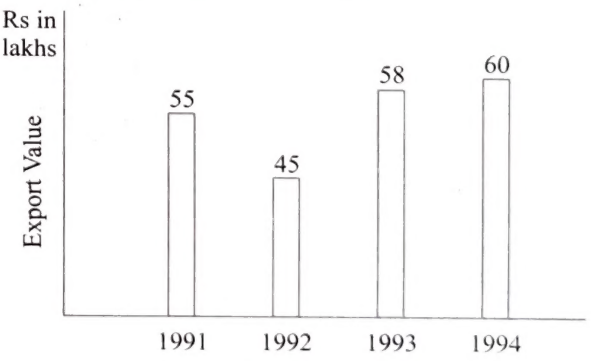


Fig. 2.2

Export value of a company X during 1991-1994

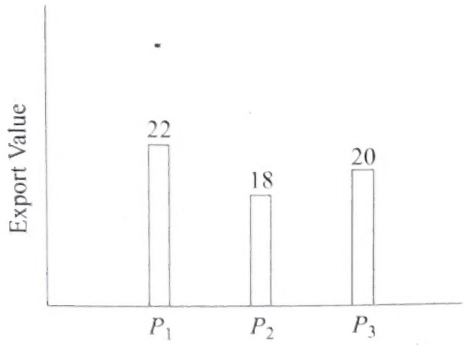


Fig. 2.3

Export value of various products P_1 P_2 & P_3 of A company X during 1994.

(b) Sub-divided Bar Chart or Component Bar Chart A sub-divided bar chart is used to represent various parts or sub-classes of total magnitude of the given variable. Sub-divided bar chart showing the monthly expenditure of two persons X & Y

A – expenditure on food

B – expenditure on education

C – expenditure on clothing

D – miscellaneous expenditure

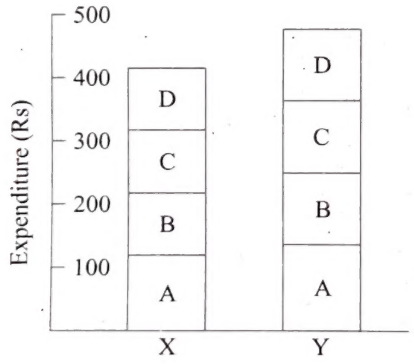


Fig. 2.4

(c) Multiple Bar Charts In this type, two or more bars are constructed adjoining each other, to represent either different components of a total or to show multiple variables.

In the adjoining bar chart three parameters of a company X have been represented for the years 1996, 1997, 1998.

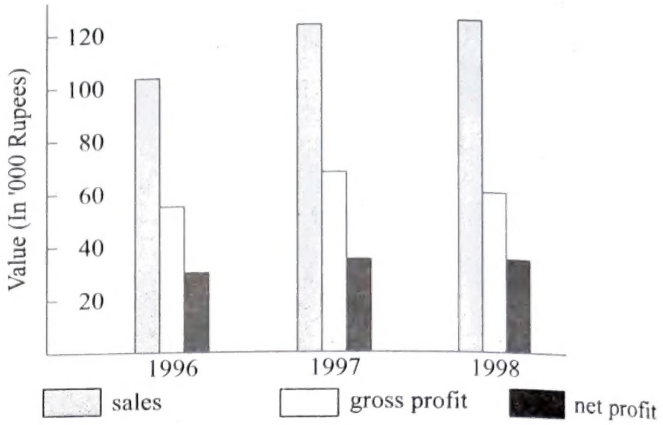
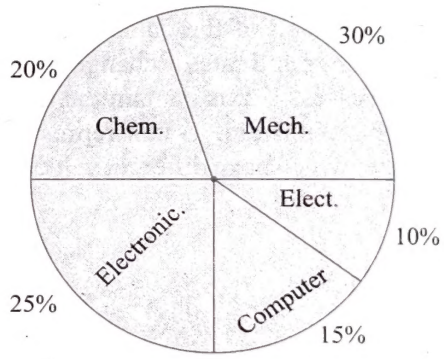


Fig. 2.5

2.2.2 Pie Charts

In this method of data representation, the total quantity is distributed over a total angle of 360° which is one complete circle or pie. Unlike the bar charts, here the data can be plotted with respect to only one parameter.



Total Engg. Students = 800

Fig. 2.6

Pie charts are useful for representing

— percentages of various elements with respect to the total quantity

— proportions of various elements with respect to the total quantity

— shares of various parties of a particular quantity

Sometimes in a pie chart, the values for various elements are represented in degrees instead of %.

Since 100% equivalent to 360°

$$\Rightarrow \qquad 1\% \approx 3.6°$$

$$\text{or} \qquad 1° \approx \frac{1}{36}\%$$

Tips: Do not convert from degree to % or vice versa unless it is specifically asked in the problem.

CHAPTER

3

Data Sufficiency

3.1 INTRODUCTION

In Data Sufficiency, a question, followed by some statements, is given. It is required to determine whether the data given in one or more statements is sufficient to answer the question. There is no need to solve the problem or arrive at an answer. It is enough to conclude that the question can be or cannot be answered.

In CSAT, two Statements S_1 and S_2 are given followed by a question which is to be checked.

There are four given choices and out of which one is correct in respect of the given two statements and the followed question.

The four choices generally given in CSAT are

(a) S_1 alone is sufficient to answer the question

(b) S_2 alone is sufficient to answer the question

(c) S_1 and S_2 are together sufficient to answer the question

(d) Either statement S_1 or S_2 alone is sufficient to answer the question,

Steps for solving Data sufficiency problems

1. Check if the first statement S_1 is sufficient to answer the given question
2. Check if the second statement S_2 is sufficient to answer the given question
3. Check if both statements are required combined to answer the question
4. Select an answer from the given options accordingly